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Cross Cultural Studies of Mathematics Teaching and Learning Recent Findings and New Directions

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"I'm sick and tired of hearing about the Japanese."
—*Comment by the Mathematics Coordinator for an affluent school district in Cook County, Illinois, as told to a cross-cultural researcher.*

The comment above is not unusual, but then neither is the opposite expression of wholehearted enthusiasm for gaining understanding of how it is that Japanese children so dramatically outperform their U.S. counterparts in tests of mathematics achievement. What is it about Asian mathematical superiority that produces such divergent responses?

If we pause to take stock of what we know about Asian mathematical performance, both responses make a good deal of sense. On the one hand, the achievement differences between Asian and U.S. children are astounding. A recent study reported by Stevenson, Lee, and Stigler (1986) compared a representative sample of fifth-grade classrooms in Sendai, Japan and Taipei, Taiwan with a sample of classrooms from the Minneapolis metropolitan area. On a test of mathematics achievement the highest-scoring U.S. classroom did not perform as well as the lowest-scoring Japanese classroom and outperformed only one of the twenty Chinese classrooms. This is a whopping difference, which leads many to be enthusiastic about learning from the Japanese.

On the other hand, although the achievement differences are large and often reported in the media, the possible mechanisms that underlie these differences are not well studied, and remain largely unknown. Teachers in the United States are often "hit over the head" with achievement differences across countries. Yet the meaning of achievement rarely has moved beyond a single score on a single test, and explanations of the differences rarely move beyond the gross level of "country." Under these circumstances, just knowing that the Japanese students are outperforming U.S. students can lead to frustration.

If we know only that the differences exist, but do not have a good description of the cultural and educational context that surrounds such differences,

it is no wonder we get sick and tired of hearing about the Japanese. What are we to make of such differences if we do not know more about what kinds of mathematical knowledge Asian children excel in learning? And how can we use such differences to inform our own practice of teaching mathematics if we know very little about how mathematics is taught in Asian schools?

The purpose of this paper is to begin to supply the research context that must surround cross-cultural studies of mathematics teaching and learning if such studies are to be useful for U.S. teachers and concerned citizens in general. We can learn from the Japanese, and from other countries as well, but only if we collect relevant information and interpret that information in a sensible way. As one author has put it (White, 1987), Japan provides us not with a blueprint for the education of U. S. children, but rather with a mirror that we can use to examine ourselves. The point of this paper is that cross-cultural studies of mathematics teaching and learning in general have the potential to provide us with just such a mirror.

The Cultural Context of Mathematics Teaching and Learning

Of all the things children learn in school, mathematics would seem to be the one subject least affected by culture. After all, numbers are numbers, and basic mathematical operations should function the same across cultures.

Although saying that "numbers are numbers" is clearly a generalization, it is the relatively transcultural nature of mathematics that makes it especially interesting for cross-cultural study. With more verbal domains of learning, it is difficult to disentangle skill from context. Not only does the context of learning vary across cultures, but so does the content of what is being learned. The nature of mathematics, on the other hand, provides the researcher with a point of commonality on which to build a comparison. The relative similarity of the body of mathematical knowledge across cultures thus makes the role of culture itself more, rather than less, prominent in understanding how children acquire mathematical knowledge, precisely because the content remains the same while the cultures vary.

In most nations around the world, mathematics is taught in school, where we find materials, activities, a teacher and a learner. (This is not to imply that mathematics is taught exclusively in school, but our primary focus here is upon mathematics learning that occurs in school.) The cultural context of mathematics learning can be described through the tools, traditions, beliefs, attitudes, and practices that support the learning of school mathematics. A brief description of these aspects of culture as they have an impact upon mathematics learning in school will be provided here.

Cultural differences are found to some degree in the tools and methods children are provided with for the performance of basic mathematical operations. Some of the most basic mathematical skills have their roots in verbal skills and thus may be influenced by cross-linguistic differences in the ways in which numbers are represented verbally. For example, recent research by Miller and Stigler (in press) has shown that learning to count is easier for children who speak Chinese than it is for English speakers, due primarily to the more consistent structure of the list of count words in Chinese. In addition, the Chinese number names of the digits 0 to 9 can be pronounced more quickly than can the English number names, thus leading to the fact that Chinese speakers can remember longer strings of numbers in a standard digit span task than can speakers of English (Stigler, Lee, & Stevenson, 1986).

Apart from the language itself, different cultures provide different representational tools for such tasks as counting and computation. Saxe (1981) and Lancy (1983) have described how peoples in Papua New Guinea use body parts as a means for counting objects, and Zaslavsky (1973) provides a rich description of indigenous African counting systems. Many Asian cultures teach children to add, subtract, multiply and divide using an abacus, or sometimes even a visual image of an abacus (Stigler, 1984; Stigler, Chalip, & Miller, 1986).

Beyond these culture-specific technologies on which some mathematical skills are constructed, there are large cultural differences in the beliefs held by parents, teachers, and children about the nature of mathematics learning. These beliefs can be organized into three broad categories: beliefs about what is *possible*, (i.e., what children are able to learn about mathematics at different ages); beliefs about what is *desirable* (i.e., what children should learn); and beliefs about what is the best *method* for teaching mathematics (i.e., how children should be taught).

Beliefs about what is possible. Cultural differences in conceptions of the possible have been found in numerous recent studies. For example, in work comparing Japanese, Chinese, and U.S. parents, it has been found that U.S. mothers are significantly more likely than Japanese mothers to believe that innate ability (as opposed to effort) underlies children's success in mathematics (Stevenson, Lee, & Stigler, 1986; Lee, Ichikawa, & Stevenson, in press). Clearly, if innate ability is believed to determine success in mathematics, then there are always limits on what one could expect a particular child to achieve in school mathematics.

Differing beliefs about what is possible also are expressed in the design of school mathematics curricula. U.S. textbooks limit children in various ways, probably because of beliefs about what is possible to teach children of different ages. For example, U.S. elementary textbooks introduce large numbers at a slower pace than do Japanese, Chinese, or Soviet textbooks, and delay the introduction of regrouping in addition and subtraction considerably longer than do books in the other countries (Fuson, Stigler, & Bartsch, in press). Although we tend to believe that it is best to introduce small numbers before large numbers, and addition without regrouping before addition with regrouping, the fact that Asian children do so well using curricula not based on such beliefs suggests that our beliefs might profitably be questioned.

Beliefs about what is desirable. Beliefs about what children should learn also differ across cultures. Although U.S. elementary schools spend far less time teaching mathematics than do schools in Japan or Taiwan (Stigler, Lee, & Stevenson, 1987), parents in the U.S. still believe that reading, not mathematics, needs more emphasis in the curriculum than it currently receives (Stevenson, Lee, & Stigler, 1986). Within the mathematics curriculum, there also may be large cross-cultural differences in beliefs about which topics should be stressed. For example, some have debated whether school mathematics should be oriented toward problem solving in the real world, or whether it should be more purely mathematical (cf. Lave, Smith, & Butler, in press).

Beliefs about the best methods for teaching mathematics. Finally, there are many cultural beliefs that relate to the proper method of instruction. Two interesting domains in this regard are beliefs about the nature of understanding, and about the role of concrete experience in children's learning. Those in the U.S., particularly with respect to mathematics, tend to assume that understanding is equivalent to sudden insight. With mathematics, one often hears teachers tell children that they "either know it or they don't," implying that mathematics problems can either be solved quickly or not at all (cf. Schoenfeld, 1985). In Japan and China, understanding is conceived of as a more gradual process, where the more one struggles the more one comes to understand. Perhaps for this reason, one sees teachers in Japan and China pose more difficult problems, sometimes so difficult that the children will probably not be able to solve them within a single class period. U.S. teachers, by contrast, tend to pose problems that will reinforce the idea that mathematics problems should be solvable in a single, insightful motion.

Related to beliefs about the nature of understanding are beliefs about the role of concrete experience. U.S. teachers tend to believe that young children need concrete experiences in order to understand, and even that concrete experiences will automatically lead to understanding. These beliefs are questioned when one observes a Japanese classroom, as discussed later in this paper. Japanese teachers apparently believe that even young children can understand abstraction and that concrete experience must be accompanied by reflection in order for understanding to follow.

Scope of this Paper

Having laid out the broader sense in which culture intrudes on the learning of mathematics, we will now narrow the scope somewhat for the remainder of the paper. While we believe that mathematics learning is always influenced by wider cultural meanings, we will restrict the rest of this paper to a discussion of cross-cultural studies of the learning of mathematics in school, in particular in modern industrialized countries where schooling is nearly universal. Thus, the focus will be on curriculum, teaching, and achievement. First, we will lay out the potential contributions that could be made by cross-cultural studies of mathematics teaching and learning in school, and then briefly review the relatively few studies that have been done. Then, we will outline the design of the University of Michigan study of mathematics learning in Japan, Taiwan, and the United States and present preliminary results from observations of first-grade mathematics classes collected as part of that study.

WHAT CAN BE LEARNED FROM CROSS-CULTURAL COMPARISON?

Before reviewing the cross-cultural studies that have been done or that are in progress, it is useful to pause first to consider both what we might expect to learn from such studies and what we cannot expect to learn. The most important thing that we should not expect to learn is the causal mechanisms that underlie high or low mathematical achievement in different cultures. For one thing, something as complex as student learning will be caused by multiple factors, and it is difficult, if not impossible, to ascertain the way in which numerous factors combine to determine achievement. In addition, the limits of comparative studies are well known (cf. Campbell & Stanley, 1963). In the absence of experimental control, the most that we can hope for are relatively well-justified hints about what is really going on.

There are, however, some very distinct advantages of cross-cultural studies. By looking closely at the teaching and learning of mathematics within the schools of different countries we can expand our own beliefs about what is possible to expect in our own culture. These new possibilities may be in terms of level of achievement, possibilities for pacing or sequencing, or new tools and methods that we were not previously aware of. Further, by observing what succeeds and what fails in other cultures, we can begin to formulate new possibilities for teaching in our own culture; we can use the natural variation that occurs across cultures to guide us in new directions, without having to explore each of those directions anew for ourselves.

When Japanese students peform as highly as they do on tests of mathematics achievement, new possibilities are opened up for the level of achievement we might think possible for U.S. students to attain. We may not have thought that it was possible or valuable for elementary school students to be formally exposed to probability and statistics. Yet, when we see Japanese children successfully learning topics such as these, we can imagine the possibility for U.S. children. In this case, we can observe behavior in Japan that cannot be seen in the U.S. This sort of variation across cultures is so striking because it allows us to realize that different

possibilities exist, not only in how something is taught, but also in whether or not something can be taught. The same can be said for the pacing and sequencing of the topics already common to Japanese and U.S. curricula. New possibilities can be imagined for how much time is necessary to teach a given topic; or we can question our assumption that topic A really must be taught before, rather than after, topic B.

New techniques for teaching can also be discovered by observing mathematics teachers in other countries. For example, simple addition and subtraction facts are generally taught by U.S. teachers either using straight memorization or, less frequently, by counting on (Fuson, 1982). In China and Japan a different technique is widespread, namely the decomposition and recomposition of numbers into groups of ten. Japanese teachers have developed TILE representations for teaching this technique (Hatano, 1982), and Chinese teachers use similar materials (e.g., strips of paper divided into ten squares with plastic flowers for filling the squares). The point is that here is a technique that is new to U.S. teachers, but that has been well-tested by Asian teachers. Cross-cultural studies allow us to benefit from the experience of a much wider range of teachers and to discover new techniques that may be usable in our own culture.

Cross cultural comparison also leads researchers and educators to a more explicit understanding of their own implicit theories about how children learn mathematics. Without comparison, we tend not to question our own traditional teaching practices and we may not even be aware of the choices we have made in constructing the educational process. For example, by observing classrooms in other cultures we can find that teachers place a large emphasis upon the whole class working together, but when observing classrooms in our own culture we find that teachers place an emphasis upon working individually. From this sort of comparative work we can gain insight into our own beliefs about how learning occurs.

In general, cross-cultural comparison allows us to observe wider variability, both in teaching and achievement, than can ordinarily be observed within a single culture. In addition, aspects of mathematics teaching that are covariant in one culture may be unconfounded in another, thus making it possible to question culture-based assumptions about the way in which two variables must be related.

A BRIEF LOOK AT PREVIOUS CROSS-CULTURAL RESEARCH

Several cross-cultural studies of mathematics learning exist. The most ambitious of these have been the studies conducted by the International Association for the Evaluation of Education Achievement (IEA) (e.g., Husen, 1967; McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987; Travers, Crosswhite, Dossey, Swafford, McKnight, & Cooney, 1985). The first IEA study (Husen, 1967), carried out in 1964, measured achievement in various mathematical topics in each of 12 different countries, at two grade levels: 8th grade and 12th grade. The second IEA study (known as SIMS, for Second International Mathematics Study) compared 17 countries in the 8th-grade component and 12 in the 12th-grade component. Both the IEA and SIMS studies measured 8th grade students' abilities to solve arithmetic, algebra, geometry, statistics, and measurement problems; and measured 12th grade students' abilities to solve algebra, geometry, elementary functions and calculus, probability and statistics, sets and relation, and number-system problems.

A major finding from both the IEA and SIMS studies was that the United States did not perform as well as had been expected. Among the countries studied, the United States consistently performed at or below the median level in each of the topic areas tested. Countries that consistently outperformed the U.S. included Japan, Hong Kong, and Belgium.

An example of the U.S.'s poor performance—and a cause for many U.S. educators to feel alarmed—can be found by examining results of the arithmetic computation subtest. The SIMS documents that, unlike most other participating countries, the U.S. is still emphasizing instruction in arithmetic in the 8th grade; however, performance in arithmetic for U.S. students is below the average for all participating countries.

The SIMS went beyond the first IEA in that it attempted to explore some of the underlying causes for the notable differences in achievement. Although the SIMS investigated many possible causes for cross-cultural differences in mathematics achievement, many of the factors were not found to relate to student achievement. For example, neither class size nor years of teacher training were related to country differences in student achievement. The authors of the SIMS report suggest that the lack of a consistent relationship between amount of formal teacher training and student achievement may be due to the fact that teachers in all countries are required to undergo some sort of formal training and are exposed to colleagues, in-service training, and other forms of continued education.

Besides ruling out uninfluential factors, the SIMS was successful at discovering two likely contributors to student mathematics achievement: the status granted to teachers in different countries (and its concomitant responsibilities and potential benefits) and the curriculum that was presented to the students. The SIMS noted that Japanese teachers are accorded far higher status than U.S. teachers and that Japanese students achieved more. The authors of the SIMS report (McKnight, et al., 1987) hypothesized that teachers' status might influence student achievement in the following way: The higher a teacher's status, the less time that she or he will be required to spend on noninstructional activities (e.g., time spent at required administrative meetings). A teacher who has fewer demands on her time will have more time to prepare for class and, thus, will presumably teach better classes. In fact, Japanese teachers were found to spend less time on administrative tasks and more time preparing for classes than U.S. teachers.

The curriculum is another variable that has been identified as influential in students' learning. The SIMS documented what they term the "implemented curriculum" by having teachers note which items on the achievement test teachers had taught to their classes. The SIMS confirms that students who are not taught calculus, for example, cannot solve calculus problems. Certainly, if students in one country have performed well or poorly on a particular type of problem, then it is useful to know whether that type of problem was or was not introduced to students.

Although the IEA and SIMS studies have covered a lot of ground in describing cross-cultural differences in mathematics achievement, other studies have also made contributions to this area. For example, Harnisch, Walberg, Tsai, Sato, & Fryans (1985) completed a large comparison of Japanese and U.S. high school students. Harnisch et al. found that Japanese students outperformed U.S. students and investigated some of the possible causes of this difference. They examined several possible contributors to achievement differences, including which mathematics courses students have completed and home background variables. The dependent variable used in this comparison was a test that measured student skill in several topic areas (including algebra, geometry, etc.). Based on several analyses, Harnisch et al., concluded, as did the SIMS, that which courses students take plays a major role in achievement differences, whereas other factors play a less influential role in mathematics achievement.

Other studies have looked at curriculum apart from achievement. For example, Fuson (Fuson, Stigler, & Bartsch, in press), and Stigler (Stigler, Fuson, Ham, & Kim, 1986) have investigated the grade levels at which addition and subtraction topics are introduced in the primary grades. Fuson et al. reported that relatively difficult addition and subtraction topics (e.g., subtraction problems that require borrowing from a zero in the subtrahend) are introduced very late in U.S. mathematics texts compared to when these topics are introduced in Soviet, Taiwanese, Mainland Chinese, and Japanese texts. Stigler et al. reported comparable findings for the introduction of addition and subtraction word problems in U.S. and Soviet texts; Soviet texts present many different types of word problems, whereas U.S. texts present only a few types of word problems. Achievement was not measured in these studies and, thus, they cannot tell us that a certain component within the curriculum has a direct influence upon a specific area of achievement. However, a more precise measurement of the curriculum, such as that provided by Fuson's and Stigler's analyses, may be valuable in exploring the exact nature of achievement differences, because previous work has already provided evidence for the link between curriculum and achievement (cf. Harnisch et al., 1985; McKnight et al., 1987).

Most previous cross-cultural work has focussed on the achievement of high school students. Only rarely have researchers attempted to trace the roots of these later achievement differences by studying younger children.

One study that has looked at younger children is reported by Song and Ginsburg (in press). This study measured mathematical skills of Korean and U.S. children at several ages, from children enrolled in day-care centers through the third grade. Song and Ginsburg found that, through the first grade, U.S. children showed higher levels of performance than Korean children, but this advantage disappeared by the second and third grades. Song and Ginsburg's results suggest that schooling may play an important role in the development of observed cross-cultural differences in mathematics achievement. However, more work is needed before we can pinpoint what in particular about schooling is causing differences in children's achievement.

We have briefly noted the range of studies that have investigated cross-cultural differences in mathematics achievement. Most of the studies have demonstrated country differences in achievement, but most have not gone far beyond the documentation of differences. What is needed is to begin to break down both the independent variable of "country" and the dependent variable of "achievement" into more fine-grained units that can deepen our understanding of both the underlying mathematical knowledge and the external cultural factors that may underlie the achievement differences.

The SIMS work on curriculum begins to analyze the components of the country effect. However, there are a great many other aspects of culture that need to be studied: teachers' and students' beliefs about mathematics learning, the role of mathematics in the everyday culture, and classroom processes involved in the teaching of mathematics are all factors that remain relatively unstudied. "Achievement," similarly, needs to be broken down into more meaningful units: Are Japanese students better at all aspects of mathematics, or only at some aspects? Are there other outcomes we might want to study as well as achievement, such as estimation skills, visual problem-solving skills, and children's future goals relating to mathematics?

The University of Michigan studies were designed to fill in some of these gaps in the cross-cultural literature. Both the first and second Michigan studies have focused on mathematics in the elementary school, an age level that has been sorely neglected in the previous research. In addition, the focus has been on those areas that have been most neglected by previous work: beliefs, attitudes, and classroom processes.

AN OVERVIEW OF THE MICHIGAN STUDIES: BACKGROUND AND DESIGN

The University of Michigan studies comparing academic achievement of children in Japan, Taiwan, and the United States began in 1978, and are presently being continued. There have been two major waves of data collection: the first in 1979-80 and the second in 1985-86. Some background on the goals of the two studies may be helpful.[1]

The First Study

The first study began with an emphasis on reading, in particular, on the role of orthography in learning to read and in the generation of reading disabilities. Although the results of the reading investigation (Stevenson, Stigler, Lucker, & Lee, 1982) proved interesting, the most striking cross-cultural differences emerged in the area of mathematics (Stigler, Lee, Lucker, & Stevenson, 1982; Stevenson, Lee, & Stigler, 1986). Although differences in mathematics achievement had been noted previously between Asian and U.S. children (e.g., Husen, 1967), never had such differences been found among elementary school children, and certainly not as early as the first grade.

The first study included only a single, individually administered test of mathematics achievement, one that had been carefully constructed for the purposes of our study. The test items focused primarily on computation (and also geometry for fifth-grade students). The test was administered to a sample of first- and fifth-grade students in Sendai, Japan; Taipei, Taiwan; and the Minneapolis metropolitan area. In each city, 10 representative schools were selected, and within each school 2 first- and 2 fifth-grade classrooms participated. Twelve children in each classroom were tested, yielding a final sample of 480 children in each of the 3 locations. The results of the test are presented in Figure 1.

Many other kinds of data were collected in the first study, including interviews with parents, cognitive testing, and classroom observations. However, it soon became clear that, given the large cross-cultural differ-

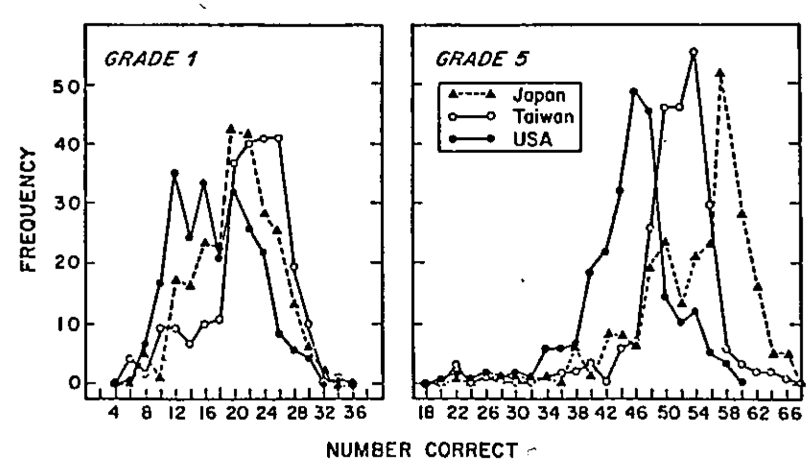

Fig. 1. Frequency distribution of scores obtained by children in Grades 1 and 5 on the mathematics test in Japan, Taiwan, and the United States. (From: Stigler, Lee, Lucker, & Stevenson, 1982.)

ences in mathematics achievement, it would be well worth our while to design a subsequent study directed specifically at understanding the cross-cultural differences in mathematics learning. We designed the study with two major goals in mind. First, we wanted to go beyond the single outcome measure produced by a standard achievement test and find out more about what kinds of mathematical knowledge Japanese, Chinese and U.S. children had acquired. Second, we wanted to probe more directly, through interviews and observations, into the teaching, curriculum, and societal context of mathematics achievement.

The Second Study

The mathematics study was again conducted in Sendai, Japan and Taipei, Taiwan. In the United States, however, we decided to use the Chicago metropolitan area instead of Minneapolis. Minneapolis is a city with a very low percentage of minority and of non-English-speaking children. The Chicago area is far more diverse in population, and thus more representative, in many respects, of mainstream United States.

In each of the two Asian cities, 10 schools were selected to participate in the study. In the Chicago area, 20 schools were chosen to represent the urban and suburban areas that make up Cook County. The decision to include twice as many schools in Cook County as in the other two locations was based on the far greater diversity of children in Cook County. Within our Chicago sample of schools, we included public and private schools; upper, middle, and lower socioeconomic status neighborhoods; predominantly Black, White, Hispanic, or ethnically-mixed schools; and urban and suburban environments. The schools were chosen in collaboration with local educational authorities to be representative of the range of schools found throughout each metropolitan area.

Near the beginning of the school year—October in Taiwan and the United States, and May in Japan—each classroom was visited by a team of testers who administered a group test of arithmetic computation and a test of reading comprehension. A total of 5524 children were tested in the three sites, spread across 160 classrooms.

Within each classroom, a subsample of 3 boys and 3 girls was randomly selected for further study. The subsample consisted of 480 children in Cook County and 240 children in each of the other two locations. The additional information on each child in the subsample was collected towards the end of the school year in which group testing had been conducted. Each child in the subsample received approximately two additional hours of individually administered mathematics tests; their mothers, teachers, and school principals were interviewed; they themselves were interviewed; and their classrooms were observed during four separate mathematics classes.

The purpose of the additional testing was to broaden our understanding of what specific knowledge differences underlie Asian superiority in mathematics achievement. There were eight mathematics tests in all, administered in two separate individual testing sessions. All of the tests were especially constructed for this study, and were judged fair by a team of researchers representing each of the cultures being studied. The tests included word problems, operations, visualization, graphing, mental calculation, number concepts, estimation, and mental image transformation.

The interviews were designed to gather information about the family and educational context in which the children's mathematical skills were developing. The "Mother Interview" included basic socioeconomic and educational data about the family and about adult and sibling involvement in the child's learning of mathematics (e.g., "How often do you help your child with his/her mathematics homework?"). In the "Teacher Interview" we obtained information about the teacher's training, about his or her beliefs on learning mathematics, and about the curriculum used (including who decides what gets taught and the teacher's opinion about that curriculum). Information regarding the entire school (including age and condition of the physical plant, number of teachers, responsibilities of teachers and students beyond teaching and learning, etc.) was obtained through an interview with the school principal.

Results from the tests and interviews are now being analyzed, and will be published soon. We will devote the remainder of this paper to analyses of the classroom observations, since this information is most directly informative about mathematics teaching in these three cultures.

MATHEMATICS TEACHING IN JAPAN, TAIWAN, AND THE UNITED STATES: SOME PRELIMINARY ANALYSES

In this section we will present findings from classroom observations conducted during both the first Michigan study, and the second, mathematics study. A full report of observations in mathematics classrooms from the first study is published elsewhere (Stigler, Lee, & Stevenson, in press). However, the method employed there, and some of the more important findings, will be reviewed to provide background for interpreting results from the second study. Methods used in the second study will be described, and very preliminary analyses of the first-grade classrooms will be reported. (A full report will follow.)

Observational Method: First Study

Each of the 120 first- and fifth-grade classrooms was visited 40 times over a two- to four-week period. The visits were scheduled to yield a stratified random sample of time across the school day and school week, thus making it possible to estimate the amount or percentage of time that was devoted to various activities. (A full description of the method can be found in Stigler, Lee, & Stevenson, 1987).

Each visit lasted about an hour, and included time for separate observations of teachers and of individual students. The procedure was to observe the target, either teacher or child, for 10 seconds, and then to spend the next 10 seconds coding the presence or absence of a checklist of categories. This procedure was repeated according to a predefined sequence that counterbalanced order of observation across the teacher and the 12 randomly chosen target students in each class. Across the two- to four-week observation period, each of the 12 children in each classroom was observed for about 33 minutes (not including coding time), and each teacher was observed for about 120 minutes.

The student coding system included 30 categories, although coding was eased somewhat by the fact that many of the categories were mutually exclusive. Various aspects of the classroom were coded from the target child's point of view, including the following: Was the class engaged in academic activities, or in transition between activities? What subject matter was being taught? How was the classroom organized and who was the leader of the child's activity? And what kinds of on- and off-task behaviors was the child engaged in.

The teacher coding system contained only 19 categories. These categories noted who the teacher was working with; what kinds of teaching behaviors the teacher was engaged in; and what kinds of feedback the teacher was offering to the students.

Specific categories from the student and teacher coding schemes will be introduced as the results are presented (although only some results from the first study will be recapitulated here). Our major emphasis will be on describing observations of first-grade classrooms completed as part of the second study.

Observational Method: Second Study

Observations for the mathematics study differed in two important ways from observations conducted in the first study. First, only mathematics classes were observed, making it impossible to compare mathematics teaching with the teaching of other subject matters. Second, in addition to an objective coding system, narrative descriptions were recorded in each class.

Each of the 160 classrooms in the mathematics study was visited four separate times over a one- to two-week period, yielding a total of 640 observations across the three locations. Observers, who were local residents of each city, arrived just before teachers began the daily mathematics lesson and observed until the mathematics class was over. Observers worked in pairs, with one observer doing the category coding and the other observer doing the narrative descriptions. The objective category coding was similar to that used in the first study and has not yet been analyzed. Thus, only analyses of the narrative observations will be presented here.

The narrative observers were instructed to write down as much as they could about what was transpiring during the class. Their goal was to record the on-going flow of behaviors and to include descriptions of all supporting materials (e.g., what was written on the blackboard, how many children were working on which problem, etc.). Observers were instructed to use a set of abbreviations common across the three countries, which enabled the observers to spend more time recording details of the class. The observers also noted, with marks in the margin, when one minute had elapsed. These minute markers were included so that we would be able to estimate the duration of various activities.

Time, Organization, and Disorganization: Findings From the First Study

The results of the first observational study served mainly to differentiate classrooms in the United States, on one hand, from classrooms in Japan and Taiwan, on the other. In the first study, very few differences emerged between Chinese and Japanese classrooms. In some respects, one only has to visit one Chinese or Japanese classroom to see vast differences between Asian and U.S. elementary school classrooms. Class size is a major difference: while the classrooms in our Minneapolis sample averaged 22 students in the first grade and 24 students in the fifth grade, the classrooms in Taipei averaged 45 and 48 students at the two grade levels, and those in Sendai, 39 at both grade levels. Most Asian classrooms are arranged with desks in rows facing the teacher, while U.S. classrooms often have desks arranged in groups.

The two dimensions on which the cultures varied most obviously were in time spent on the teaching and learning of mathematics and in the level of organization in the classroom.

Time. Children in Japan and Taiwan spend significantly more time in school than do children in the United States, and this ultimately translates into Japanese and Taiwanese children spending significantly more time learning mathematics. School is in session 240 days per year in both Japan and Taiwan, compared to only 180 days per year in the United States. Although first-graders in all three cities that we studied spent about 30 hours per week in school, fifth-graders in Sendai spent 37 hours a week in school, those in Taipei, 44 hours, and those in Minneapolis, still only 30 hours.

Although we observed only during academic classes, and not during such periods as lunch, gym, recess, or assemblies, the students, nevertheless, were not always engaged in academic activities. In first grade, U.S., Chinese, and Japanese children spent 69.8%, 85.1%, and 79.2% of the time, respectively, engaged in academic activities. At the fifth grade the corresponding percentages were 64.5%, 91.5%, and 87.4%. At both grade levels, Chinese and Japanese children spent a much higher percentage of their time engaged in academic activities than did U.S. children. Furthermore, although the percentage of time spent in academic activities increased between first and fifth grade for the Asian children, the percentage actually declined slightly across grade levels for the U.S. children.

Our observers recorded the percentage of time devoted to different subject matters. The majority of time in all three cultures was devoted to either reading/language arts or to mathematics. Although the total percentage of time devoted to either one of these two subject matters was similar across the three cultures, the way in which time was apportioned between the two varied significantly by culture. As is apparent in Figure 2, U.S. teachers at both grade levels devoted more time to reading/language arts and less time to mathematics than did Chinese and Japanese teachers. By the fifth grade, both Chinese and Japanese teachers spent approximately equal amounts of time teaching mathematics and reading. U.S. teachers, by contrast, spent almost three times as much time on reading as they did on mathematics.

Calculations based on the hours per week spent in school, the percentage of time spent in academic activities, and the percentage of time those academic activities were mathematics, versus reading/language arts, allow us to estimate the number of hours each week children in the three cultures spend working on the different subject matters. The results of these calculations are presented in Table 1. The cross-cultural differences in the number of hours devoted to mathematics instruction are large—sufficiently large, in fact, that they could go a long way toward explaining the cross-cultural differences in mathematics achievement.

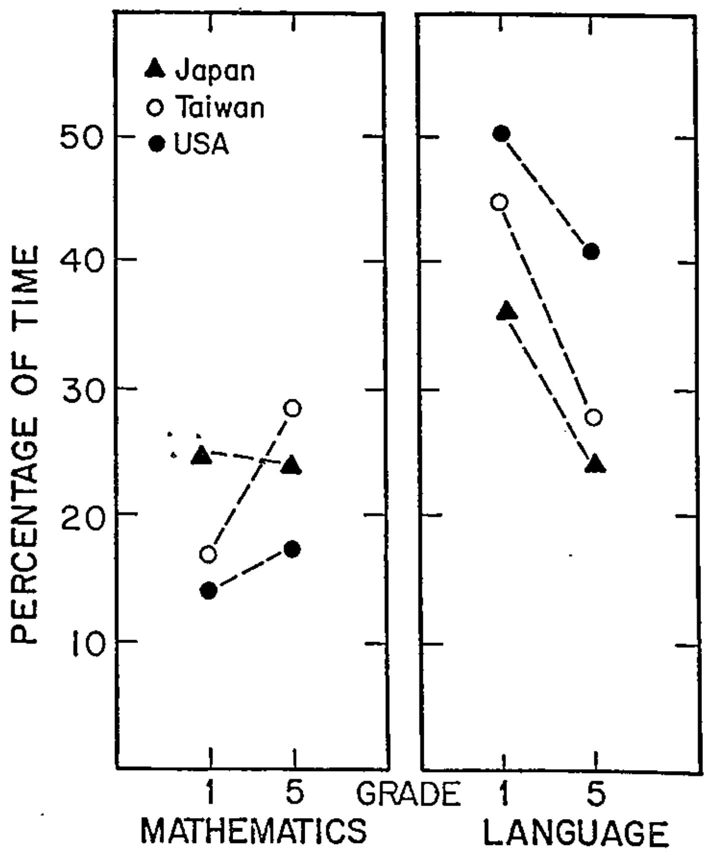

Fig. 2. Percentage of time spent teaching mathematics in Japan, Taiwan, and United States first- and fifth-grade classrooms. (From Stigler, Lee, & Stevenson, in press.)

Level of Organization. The second dimension that differentiated U.S. mathematics classrooms from those in Japan and Taiwan was the level of organization apparent in the classroom. Classrooms in Japan and Taiwan were highly organized and orderly; those in the United States more disorganized and disorderly. These differences were indicated in the coding system in various ways.

Three sets of categories dealt with the way in which the classroom was organized during mathematics instruction. In one set, observers coded whether the target child was working as an individual, as part of a small group, or as part of the whole class. The second set coded similar information, but from the point of view of the teacher: Was the teacher working with the whole class, a small group, an individual, or no one at the time of the observation? In the third set of categories, observers coded who was the leader of the activity in which the target child was engaged: the teacher or no one.

The results of both student and teacher observations regarding the unit of organization (whole class, group, or individual) are presented in Figure 3. Japanese and Chinese students spent the vast majority of their time

TABLE 1

Number of Hours Each Week Spent in Language Arts and Mathematics

	Country		
	U.S.A.	Taiwan	Japan
Mathematics			
Grade 1	2.9	3.9	6.0
Grade 5	3.4	11.4	7.6
Language Arts			
Grade 1	10.6	10.5	8.8
Grade 5	8.2	11.2	7.8

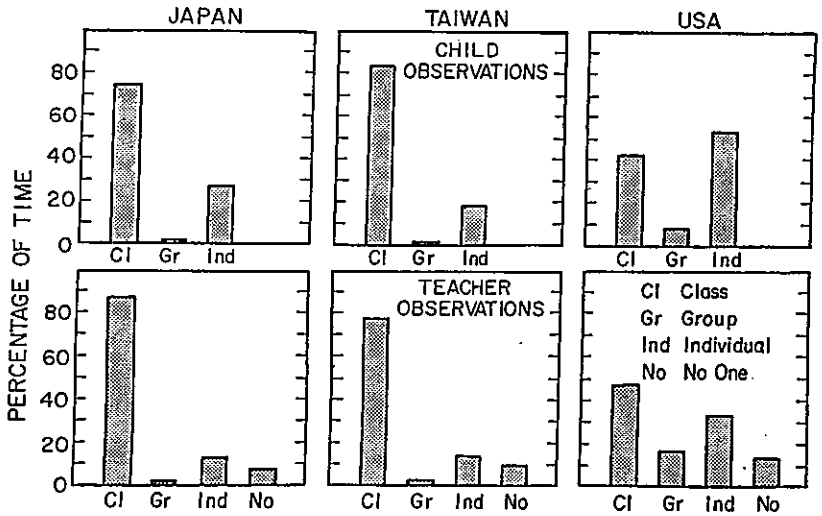

Fig. 3. Percentage of time (a) children spent working as a member of the whole class, a small group, or as an individual (top panel); and (b) teachers spent working with the whole class, a small group, an individual student, or no one (bottom panel). (From Stigler, Lee, & Stevenson, in press.)

working, watching, and listening together as a class, and were rarely divided into smaller groups. U.S. children, by contrast, spent the majority of their time working on their own, and a smaller amount of time working in activities as members of the whole class. The same picture emerges when teachers are observed (the lower panel of Figure 3). U.S. teachers spent more time working with individuals and less time working with the whole class than did Chinese or Japanese teachers. In addition, U.S. teachers were coded in mathematics classes as working with no students 13% of the total time, as opposed to only 6% of the total time for Japanese teachers and 9% of the total time for Chinese teachers.

The counterpart to these findings is displayed in Figure 4, where we see what percentage of the total time in mathematics classes students were part of a teacher-led activity, and what percentage they were part of an activity with no leader. In Taiwan, the teacher was the leader of the children's activities 90% of the time, as opposed to 74% of the time in Japan and only 46% of the time in the United States. No one was leading the student's activity 9% of the time in Taiwan, 26% of time in Japan, and 51% of the time in the United States.

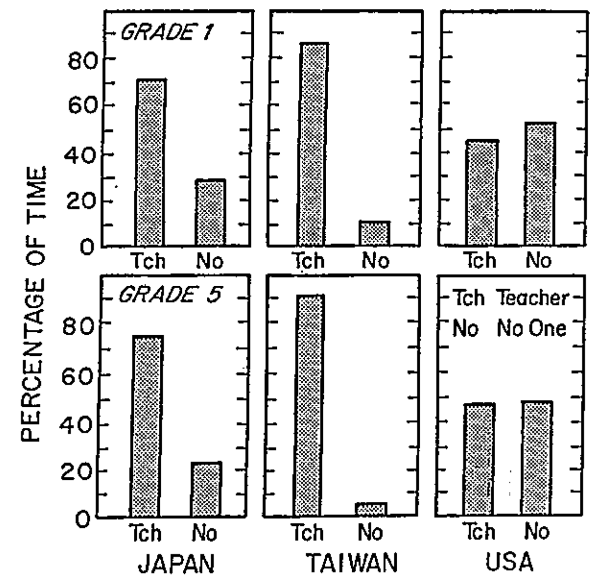

Fig. 4. Percentage of time students spent in activity led by the teacher and by no one. (From Stigler, Lee, & Stevenson, 1987.)

Taken together, these findings indicate that classrooms in the Asian cultures are more hierarchically organized, with the teacher directing her energies to the whole class, and with students more often working under the direct supervision of the teacher. Because of these differences in organization, U.S. students experience being taught by the teacher a much smaller percentage of time than do the Asian students, even though U.S. classes contain roughly half the number of students.

In addition to the relative disorganization that characterizes U.S. classrooms, there is a relative disorderliness as well. This disorderliness was picked up in our coding system by a set of categories for coding the incidence of inappropriate or off-task student behaviors. If the target child was not doing what the teacher expected him or her to do, he or she was judged as being off-task. Two categories of off-task behaviors were distinguished: those behaviors involving inappropriate peer interaction, and those the target child engaged in alone. In addition, we coded whether or not the target child was out of his or her seat. The results from these observations are presented in Figure 5.

There were large cross-cultural differences in the overall percentage of

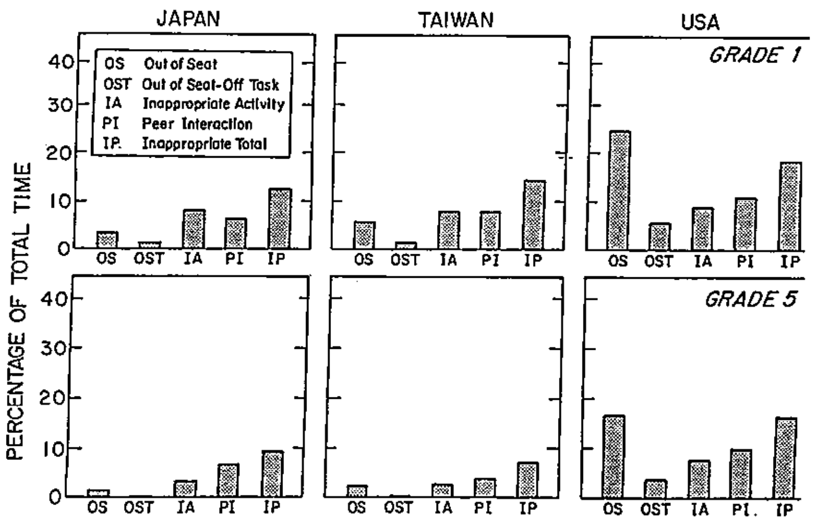

Fig. 5. Percentage of time students in the three countries were coded as engaged in various off-task activities. (From Stigler, Lee, & Stevenson, in press.)

time students spent engaged in inappropriate, off-task activities. Across both grade levels, during mathematics class U.S. students were off-task 17% of the time, as opposed to only 10% of the time for Chinese and Japanese students.

U.S. students were coded as being out of their seats during mathematics classes 21% of the time, whereas Chinese and Japanese children were out of their seats 4% and 2% of the time, respectively. Of course, students being out of their seats does not necessarily imply that they are off-task, particularly in U.S. classrooms. However, if we look at the percentage of time students were *both* out of their seats *and* off-task, the American percentage was 5 times as high as that in the other two countries (5% versus less than 1% in Japan and Taiwan).

Coherence and Reflectivity: Preliminary Ideas from First-Grade Narrative Observations

The data derived from the first observational study is informative in some respects; indeed, we get a clear picture of the frequency with which classrooms are organized in various ways, and we get basic information about

how time is spent by students in the three countries. However, even though the observations reported above were made in mathematics classes, we learned very little about how mathematics is actually taught in the three cultures. The narrative observations collected in the current study provide us with richly detailed information concerning what happens in mathematics classes in Sendai, Taipei, and Chicago. We are only beginning our analyses of these data, and will present some early hunches based only on observations of first-grade classrooms. To anticipate what these hunches are, we find two important dimensions along which the classrooms in the three cultures vary.

The first is coherence, from the child's point of view. We will argue, for now at least, that both Chinese and Japanese classrooms provide more opportunities than U.S. classrooms for the students to construct a coherent representation of the sequence of events that make up a typical mathematics class and to understand the goals of the activities in which they are engaged. The second dimension is one that ranges from an emphasis on performance, on the one hand, to an emphasis on reflection and verbalization, on the other. On this dimension, the classrooms in Taiwan tend toward the performance end, and those in Japan toward reflectivity. U.S. classrooms appear confused in this regard, and accomplish neither goal well.

We will first present some information regarding the way in which the narrative observations are being coded. Then we will turn to a fuller account of how we have arrived at the dimensions of coherence and reflectivity as significant and interesting ways to differentiate between classrooms in the three cultures.

Coding. The rich nature of the data gathered in narrative observations exacts its cost later when it is necessary to code the data. We were faced with 640 different narrative descriptions of mathematics classes, in three different languages. Not all observations were of equal quality; in every location, some observers recorded more detail than others, and some were more consistent in their use of abbreviations. How were we to code and summarize the data into a form that would be useful in characterizing cross-cultural differences in mathematics teaching?

We first convened a group of bilingual and trilingual coders to simply read all of the observations. These coders spent weeks reading observations and summarizing their contents in English for the other members of the research group. In addition, a subset of the observations were translated verbatim into English. In this way, we developed a feel for the range of situations we would have to code and some intuitions about cross-cultural differences that would be worth coding. We decided that we wanted some predefined categories that we could apply to the observations, but that we also wanted to preserve a great deal of the detail so that it could be further analyzed later. The coding system we constructed represented a balance of all of these needs.

We decided to begin the coding process by dividing each observation into segments, which would be our basic unit of analysis. We found that, as we read through the descriptions of classes, it was relatively easy to divide the class into natural-seeming segments and that we had relatively high agreement amongst group members about where to make the divisions. We gradually developed a more explicit definition of segments by attending to the conditions under which we would say that the segment had changed. A segment was defined as changing if there was a change in *either* topic, materials, or activity.

Topics were globally defined, including categories such as telling time, measurement, or addition facts. Materials included such items as textbooks, worksheets, the chalkboard, or flashcards. Activities, again, were rather molar: examples included seatwork, students solving problems on the chalkboard, or teachers giving explanations. All categories were inductively derived based on our first pass through the data, and we felt that the categories we developed were sufficient to describe the classrooms from the three cultures. The categories were not intended as the full description of the class, but rather as a way of organizing the information into a more useful format.

In addition, an English-language summary was constructed of each segment that would convey in some detail what was going on during the segment. The summary was not intended to be a translation, but rather a briefer recapitulation of the contents of the observation. A great deal of detail still was maintained, however, such as direct quotes from the teachers and students as they participated in the mathematics class. Summaries were written in narrative form so that their contents could be examined later by English speakers not fluent with Chinese or Japanese. The summaries were standardized somewhat by the use of keywords that would serve to alert us to the presence or absence of certain categories in the classroom. For example, whenever a student was observed asking a question to the teacher, the summary would include the standard keyword "S-to-T" so that a computer search for all such situations would be facilitated. Our goal was to make the summaries as consistent as possible in their style and language.

Coherence. After reading the corpus of first-grade observations, all of us in the coding group were struck with the sense that the Chinese and Japanese classes provided more opportunities for the students to construct a coherent account of the sequence of events and activities that make up a mathematics class. In other words, it appeared to us that the Chinese and Japanese classes were in some sense more comprehensible than were the U.S. classes.

The meaning we attached to the term *coherence* is similar to that used in the literature on story comprehension. Of particular relevance is work by Stein and her colleagues on defining the components of a well-formed story (e.g., Stein, in press; Stein & Policastro, 1984) and work by Trabasso on the role of coherence in story comprehension (e.g., Trabasso & van den Broek, 1985). A well-formed story, which also is the most easily comprehended, consists of a protaganist, a set of goals, and a sequence of events that are causally related to each other and to the eventual realization of the protaganist's goals. An ill-formed story, by contrast, might consist of a simple list of events strung together by phrases such as "and then . . .," but with no explicit reference to the relations among events. The important point is that ill-formed stories are particularly difficult to comprehend, and even more difficult for children to comprehend than for adults. Thus, a certain amount of coherence in input is required if the listener is to be able to construct a coherent representation of the story.

The analogy between a story and a mathematics classroom is not perfect, but it is close enough to be useful for thinking about the process by which children might construct meaning from their experiences in mathematics class. A mathematics class, like a story, consists of sequences of events related to each other and, hopefully, to the goals of the lesson. What we tend to find in the U.S. classroom observations, unfortunately, are sequences of events that go together much like those in an ill-formed story. If it is difficult for adult observers to construct a coherent representation of the events that constitute a first-grade mathematics class, then it surely would be impossible for the average six-year-old to do so.

What are some of the devices employed by Japanese and Chinese teachers to provide more coherence across the events that constitute a mathematics class? One of the major devices we have found is the tendency in Japan and Taiwan to spend an entire 40 minute mathematics class period on the solution of only 1, 2, or 3 problems. A problem thus, in a sense, serves as the protaganist that runs as a single thread through the story, a natural link to tie different segments together.

This devotion of an entire class period to a single problem would seem excessive to U.S. educators. In no class did we observe a U.S. teacher sticking with a single problem for so long, and, indeed, it appears that U.S. teachers value just the opposite approach. In recent research that examined characteristics of expert mathematics teachers in the United States, it was reported that the expert elementary mathematics teacher can get through 40 problems in a single class, whereas the novice teacher may only cover 6 or 7 problems (Leinhardt, 1986; Leinhardt & Greeno, 1986). It would appear that Japanese or Chinese teachers are striving for a different goal. Or, perhaps they are just adapting to a different reality; the value placed on homework in both of these Asian cultures means that repetitive practice can be accomplished at home and class time can be reserved for teaching. U.S. teachers must, especially at the first-grade level, accomplish both purposes during the school day.

It is important to note that this does not mean that Japanese and Chinese classrooms are boring for lack of variety. Variety as indexed by change in segment is approximately equal across the three cultures: The typical first-grade mathematics class in all three cities consists of 5 or 6 segments, each lasting 7 or 8 minutes. What is different is the nature of the changes that occur from one segment to another. While in Japanese and Taiwanese classrooms segment changes are more often coded because of changes in materials or activities, without a change in topic being taught, in U.S. classrooms the changes are more often coded because of a change in topic being taught (also see Berliner & Tikunoff, 1976). In Japan, only 6.9% of segment changes are marked by changing topic, in Taiwan, 16.1% and in the United States, 24.8%.

Remember that a change in topic does not mean merely a change in problem, but rather a change on the order of, say, starting with measurement and moving to multi-digit addition. For example, one first-grade U.S. class started with a segment on measurement, then proceeded to a segment on simple addition, then to a segment on telling time, and then to another segment on addition. The whole sequence was called "math class" by the teacher, but it is unclear how this sequence would have been interpreted by a child. In this case, it seems that it would be impossible for anyone to construct a coherent account of the whole class.

In other cases, the sequence itself *could* be construed coherently, but U.S. teachers do little to help the child construct a coherent representation. A good example of this kind of situation is provided by the topic of measurement as it is normally taught in first-grade classrooms. Most U.S. textbooks teach fundamental measurement in the following sequence: First they teach children to compare quantities directly, and to say which is longer, wider, and so forth. Next, nonstandard units of measurement are introduced, and children are taught to ascertain, for example, how many paper clips long their pencils are. Finally, students are introduced to the concept of standard units and taught to measure objects in inches or in centimeters. This is a sensible sequence and could conceivably be taught in a coherent manner.

Let us examine the way in which this sequence is implemented in one U.S. classroom in our sample. In the first segment, the teacher has children examine objects (pencils, crayons, paper clips, chalk, etc.) to determine which are longer. The teacher then moves the class to the next segment, and the following quotation begins at the point of transition:

> OK, open your workbooks to page 12. I want you to measure your desk in pencils, find out how many pencils it takes to go across your desk, and write the answer on the line in your workbooks. [Children carry out instructions.] Next see how many paper clips go across your desk, and write that number next to the paper clip in your workbook. [Children continue to follow instructions.] OK, the next line says to use green crayons, but we don't have green crayons so we are going to use blue crayons. Raise your hand if you don't have a blue crayon. [Teacher takes approximately 10 minutes to pass out blue crayons to students who raise their hands; coded as a transition segment.] Now write the number of blue crayons next to the line that says green crayons. [Teacher then moves on to the third segment.] OK, now take out your centimeter ruler and measure the number of centimeters across your desk, and write the number on the line in your workbooks.

What is fascinating about this particular class is that there is absolutely no marking by the teacher of the transition points—the three segments just follow each other as though there were no transition. There is no discussion of how each exercise is important in providing students with an understanding of measurement: no discussion of why units are important, or why standard units are important; no discussion of the historical development of measurement procedures that could provide more meaning to the sequence of activities; and no discussion of the goals of the class, and how each activity relates to those goals. More time is devoted to making sure students have a blue crayon, which is totally irrelevant to the purpose of the lesson, than to conveying the purpose of the three segments on measurement.

If we put ourselves in the child's position, what is the likelihood that we would construct a coherent, meaningful account of this particular class? Although each of the three measurement activities is intended to provide some new insight within the context of the preceding activity, the rationale behind the sequence is not made clear to the students. As someone has remarked, it is like giving children the punchline without the joke[2]: Ideally, the students would say "I get it!" as each new segment unfolds. But can they "get it" without some explicit reference to the links that tie the segments together? It is highly unlikely, especially for a group of six-year-olds.

In Chinese classrooms, and in Japanese classrooms to an even greater extent, we see teachers providing explicit markers to aid children in inferring the coherence across different segments within a lesson, and across different lessons. Rarely is the logical flow of an Asian first-grade class broken to pursue irrelevant business (such as passing out blue crayons) that may give students the wrong ideas about what is important about mathematics. Transitions often are marked by verbal discussion of the relation between two segments, and classes, especially in Japan, often start with the teacher explaining the goal of the day's class and how the activities relate to the goal.

One Japanese first-grade teacher was quoted as asking this question to a student at the beginning of a mathematics class: "Would you explain the difference between what we learned in the previous lesson, and what you came across in preparing for today's lesson?" To hear a question of this sort posed to a six-year-old would be surprising to most U.S. educators. Perhaps more surprising is that the student was able to answer the question. This kind of interchange highlights the attention paid in Asian classrooms to the students' conscious construction of a coherent account of the classroom experience.

Reflectivity. The second dimension we will discuss is reflectivity: Classrooms can vary in the degree to which they emphasize performance and practice, on one hand, versus reflective thinking and verbalization, on the other. This is an interesting dimension because it appears to differentiate Chinese classrooms from Japanese classrooms, with the Chinese classrooms being more performance oriented and the Japanese classrooms more reflective. U.S. classrooms do not seem to take a definite stand on this dimension. They are not at all reflective, like the Japanese classrooms, nor do they place a consistent emphasis on performance, like the Chinese classrooms. As we will show, at times they attempt both, but end up more often in confusion.

Perhaps a good way to begin to understand this dimension is through looking at verbalization. One index we have of verbalization is through our coding of the incidence of explanations, either by teachers (EXP-T) or by students (EXP-S). EXP-T or EXP-S could be coded as the main activity that characterizes a segment, or as summary keywords describing transitory events embedded within other segments. The incidence of explanations is presented in Table 2.

There are large differences between Japan and the other two cultures in the incidence of verbalization. We have not yet carried out the formal analyses, but it appears from our reading of the observations that verbalization is often used by Japanese teachers as a means of relating different activities to each other and as a means of discussing the principles that underlie different mathematical procedures. The high incidence of verbal explanation in the Japanese classrooms is especially interesting, given that we are observing first-grade classrooms. From the U.S. point of view, actions, not

TABLE 2

Incidence of Explanations in Japanese, Chinese, and U.S. Classrooms

	Japan	Taiwan	U.S.A.
Percentage of Segments:			
Teacher Explains (EXP-T)	18.0	.3	4.0
Student Explains (EXP-S)	5.0	.3	.1
Percentage of Summaries:			
Containing EXP-T	40.0	13.0	16.0
Containing EXP-S	15.0	10.0	4.0

words, are supposedly more successful means of communicating with a six-year-old.

The Japanese emphasis on reflectivity is further illustrated by comparing statements made by Japanese and Chinese teachers. The Chinese teachers emphasize getting the right answer quickly, whereas Japanese teachers often tell students that the answer is unimportant. Japanese teachers stress the process by which a problem is worked and exhort students to carry out procedures patiently, with care and precision. The Chinese teachers emphasize "do," the Japanese teachers, "think." In fact, the word "think" appears in our Japanese protocols more than twice as frequently as it does in either the Chinese or U.S. protocols.

The Chinese emphasis on speed and on getting the answer is evidenced by the following excerpts from the observations:

- Teacher hands out an April calendar, and instructs students to fill in the missing dates as quickly as they can.
- Teacher runs a competition for "speeded" mental calculation, and writes students' names on the board in order of their speed.
- Teacher evaluates students' blackboard work with a "check" if it was correct *and* fast, and an "X" if it was *either* incorrect or slow.

The Japanese emphasis on reflection and verbal discussion is illustrated in the following excerpts from observations:

- Teacher leads a discussion with students on "which is the best method" for solving a particular problem.
- Teacher directs students' attention to a list of numbers on the blackboard and asks them to look for patterns: "What do you notice here?
- Teacher writes a word problem on the blackboard and tells students the problem is to come up with an equation that can be used to solve the problem. Teacher asks students to choose a partner, and "think about it together in pairs".
- Teacher puts the problem 30+60 on the blackboard, and tells students she wants them to "think about the problem for a whole minute" before beginning to solve it.
- Teacher has one student solve a problem on the blackboard. When the student is finished, he turns to the class and says, "Am I correct?" The class answers, "Not exactly . . .", and then proceeds to correct him.
- The teacher says: "The answer is 41, but that is not as important as the method by which you get it. The crucial thing is the right way to getting the answer."

It is hard to determine where U.S. classrooms fall on this dimension of reflectivity, partly because of the relative disorganization alluded to earlier. It is our impression that U.S. teachers are giving mixed messages to students and confusing them about the goals of mathematics. While Japanese and Chinese teachers are communicating consistent messages about what they view as the most important goal of mathematics instruction (i.e., performance or reflective understanding), U.S. teachers make statements that we find confusing with respect to determining the goals of instruction. U.S. first-grade teachers rarely produced the type of statement that would encourage reflection about mathematics and instead tended to produce statements that were misleading about the goals of mathematics. Here are some examples of statements made by U.S. teachers in our sample:

- On the speed/accuracy issue: "Speed is not as important as neatness."
- "I'm giving you another chance to correct your papers so you don't get any red 'X's on your work."
- Teacher explains: "Let's think about counting by 2's . . . what you do is skip a number, say a number, skip a number, say a number, etc."
- Teacher queries whole class: "What is the rule for subtraction?" Class responds chorally: "The big number goes first."

These are anecdotes, and we must await more careful analysis before making statements about how representative they are of teachers in the three cultures. However, we do not believe we have misled the reader by the particular quotes we have chosen to report. Obviously, there are some superb teachers in our U.S. sample. But the statements we have chosen do, we believe, represent a significant amount of what we see in first-grade classrooms. If we were the students in these classes, what would we construe as the nature and goals of mathematics? This, it seems, is a question worth pursuing.

CONCLUSION

We began this paper with a general discussion of mathematics teaching and learning in its cultural context. We then narrowed our focus to school mathematics and described the kinds of cross-cultural studies that have been done on the teaching and learning of school mathematics. We then launched into a more detailed description of the University of Michigan studies. We ended with some preliminary analyses of narrative observations of first-grade mathematics classrooms in Japan, Taiwan, and the United States, collected as part of the second Michigan study.

An important point, in our opinion, is the amazingly small number, and relatively narrow scope, of cross-cultural studies that have been done. The only major effort aside from the Michigan studies has been the work of the IEA and SIMS. The IEA and SIMS studies have been ground breaking in their analyses of curriculum and achievement, but have not pursued some of the more important cultural factors that surround the teaching of mathematics, nor have they examined student outcomes other than achievement.

What have been particularly lacking, in our opinion, are studies of how

mathematics is taught in classrooms in different cultures. It is for this reason that we devoted the greatest part of the paper to reporting analyses of classroom observations from the Michigan studies. As we discussed earlier, there are many aspects of culture that are brought to bear in the teaching and learning of mathematics: beliefs, attitudes, practices, tools, and traditions. There can be no doubt that what happens in the classroom is in some sense a reflection of the wider society in which the classroom exists. Nevertheless, if we want to reform mathematics teaching, it seems that the classroom is a good place to start. Although it is difficult to change what happens in classrooms, it is far more difficult to change broader aspects of the culture.

It is important to emphasize that findings from our observations, or from any other studies that may be done in the future, are not meant as an indictment of U.S. teachers. Indeed, one of the more striking aspects of our findings is the difficult challenge U.S. teachers face each day as they enter their classrooms. Few would want to be in their shoes. We must get beyond the tendency to assign blame if we are to make maximum use of what can be learned from cross-cultural studies of mathematics teaching and learning. There is a great deal that we can learn about ourselves by carefully observing others. We hope others are encouraged to do cross-cultural studies and to deal with the difficult issues of interpretation that inevitably arise. The knowledge that can be gained is worth the difficulty.

NOTES

[1]The Michigan studies have been directed by Professor Harold Stevenson at the Center for Human Growth and Development at the University of Michigan, and conducted in collaboration with numerous colleagues (in addition to the authors of this paper): Shin-ying Lee at the University of Michigan; Chen-chin Hsu at National Taiwan University Medical College; Lian-wen Mao of the Taipei Bureau of Education in Taiwan; and Seiro Kitamura, S. Kimura and T. Kato of Tohoku Fukushi College in Sendai, Japan. The first study was supported by NIMH grants MH 33259 and MH 30567. The second study was supported by the National Science Foundation.

[2]We are indebted to Richard A. Shweder for this analogy.

REFERENCES

Berliner, D. C. & Tikunoff, W. J. (1976). The California beginning teacher evaluation study: Overview of the ethnographic study. *Journal of Teacher Education*, *27*(1), 24-30.

Campbell, D. T. & Stanley, J. C. (1963). *Experimental and Quasi- experimental designs for research*. Chicago: Rand McNally.

Fuson, K. C. (1982). An analysis of the counting on procedure in addition. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 67-81). Hillsdale, NJ: Erlbaum.

Fuson, K., Stigler, J., & Bartsch, K. (in press). Grade placement of addition and subtraction topics in Japan, Mainland China, the Soviet Union, Taiwan, and the United States. *Journal for Research In Mathematics Education*.

Harnisch, D., Walberg, J., Tsai, S-L, Sato, T., & Fryans, L. (1985). Mathematics productivity in Japan and Illinois. *Evaluation in Education*,*9*, 277-284.

Hatano, G. (1982). Learning to add and subtract: A Japanese perspective. In T.P. Carpenter, J.M. Moser, & T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 211-223). Hillsdale, NJ: Lawrence Erlbaum Associates.

Husen, T. (1967). *International study of achievement in mathematics.* NY: Wiley.

Lancy, D. F. (1983). *Cross-cultural studies in cognition and mathematics.* New York: Academic Press.

Lave, J., Smith, S., & Butler, M. (in press). Problem solving as everyday practice. In E. Silver & R. Charles (Eds.), *Teaching and evaluation of mathematical problem solving.* Reston, VA: National Council of Teachers of Mathematics.

Lee, S., Ichikawa, V., & Stevenson, H. (in press). Beliefs and achievement in mathematics and reading: A cross-national study of Chinese, Japanese, and American children and their mothers. In M. Maehr (Ed.), *Advances in motivation: Vol. 7.* Greenwich, CT: JAI Press.

Leinhardt, G. (1986). Expertise in math teaching. *Educational Leadership, 43*(7), 28-33.

Leinhardt, G. & Greeno, J.G. (1986). The cognitive skill of teaching. *Journal of Educational Psychology, 78*(2), 75-95.

McKnight, C., Crosswhite, F., Dossey, J., Kifer, E., Swafford, J., Travers, K., & Cooney, T. (1987). *The underachieving curriculum: Assessing U.S. School mathematics from an international perspective.* Champaign, IL: Stipes.

Miller, K. & Stigler, J. (in press). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive Development.*

Saxe, G. B. (1981). Body parts as numerals: A developmental analysis of numeration among the Oksapmin in Papua New Guinea. *Child Development, 52, 306-316.*

Schoenfeld, A. H. (1985). Mathematical problem solving. New York: Academic Press.

Song, M. J. & Ginsburg, H. P. (in press). The development of informal and formal mathematical thinking in Korean and U.S. Children. *Child Development.*

Stein, N. L. (in press). The development of children's storytelling skills. In M.B. Franklin & S. Barten (Eds.), *Child language: A book of readings.* New York: Oxford University Press.

Stein, N. L. & Policastro, M. L. (1984). The concept of a story: A comparison between children's and teachers' perspectives. In H. Mandl, N. Stein, & T. Trabasso (Eds.), *Learning and comprehension of text* (pp. 113-155). Hillsdale, NJ: Lawrence Erlbaum Associates.

Stevenson, H. W., Lee, S. Y., & Stigler, J. W. (1986). Mathematics achievement of Chinese, Japanese, and American children. *Science, 231*, 693-699.

Stevenson, H. W., Stigler, J. W., Lucker, G. W., and Lee, S. Y. (1982). Reading disabilities: The case of Chinese, Japanese, and English. *Child Development, 3* 1164-1182.

Stigler, J. W. (1984). "Mental Abacus": The effect of abacus training on Chinese children's mental calculation. *Cognitive Psychology, 16*, 145-176.

Stigler, J., Chalip, L. & Miller, K. (1986). Consequences of skill: The case of abacus training in Taiwan. *American Journal of Education, 94*(4), 447-479.

Stigler, J., Fuson, K., Ham, M., & Kim, M. (1986). An analysis of addition and subtraction word problems in U.S. and Soviet elementary mathematics textbooks. *Cognition and Instruction, 3*(3), 153-171.

Stigler, J. W., Lee, S. Y. & Stevenson, H. W. (1986). Digit memory in Chinese and English: Evidence for a temporally limited store. *Cognition, 23*, 1-20.

Stigler, J. W., Lee, S. Y. & Stevenson, H. W. (in press). Mathematics classrooms in Japan, Taiwan, and the United States. *Child Development.*

Stigler, J. W., Lee, S. Y., Lucker, G. W., & Stevenson, H. W. (1982). Curriculum and achievement in mathematics: A study of elementary school children in Japan, Taiwan, and the United States. *Journal of Educational Psychology, 74*(3), 315-322.

Trabasso, T. & van den Broek, P. (1985). Causal thinking at story comprehension. *Memory and Language, 24*, 612-630.

Travers, K. J., Crosswhite, F. J., Dossey, J. A., Swafford, J. O., McKnight, C. C., & Cooney, T. J. (1985). *Second international mathematics study summary report for the United States.* Champaign, IL: Stipes.

White, M. (1987). *The Jananese educational challenge: A commitment to children.* New York: Free Press.

Zaslavsky, C. (1973). *Africa counts.* Boston, MA: Prindel, Weber, & Schmidt.

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