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Gender Differences in Elementary School Children's Strategy Use and Stragegy Preferences on Multidigit Additions and Subtraction Story Problems

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Abstract

Gender differences in the strategies elementary school children use to solve multidigit addition and subtraction story problems that require regrouping are investigated in two studies. Study 1 replicates the Fennema and colleagues (1998) study by reexamining previously published data on 72 children's addition and subtraction solution strategies. Study 2 extends previous findings by identifying the reasons for 70 third-grade children's strategy preferences. Gender differences in strategy use, although significant, were not as strong as those reported by Fennema and colleagues (1998). But the reasons boys and girls gave for choosing particular strategies suggests that the sources of these differences might be important and there are different understandings of what it means to learn and do mathematics. Results support the hypothesis that early gender differences in strategy use influence later gender differences in problem solving performance that appear in middle school and continue throughout high school.

Research on gender differences in mathematics reveals that as children get older, more pronounced differences appear in the performance of boys and girls depending on the nature of the tasks. More specifically, it has been shown that girls outperform boys on computation tasks in elementary school (Armstrong, 1981; Fan, Chen, & Matsumoto, 1997; Fennema, 1974; Fennema & Carpenter, 1981; Fennema & Sherman, 1978; Hyde, Fennema,

& Lamon, 1990; Marshall 1984; Marshall & Smith, 1987). However, a transition occurs in middle school at which time boys begin to outperform girls on problem solving tasks. Although the size of these differences in mathematics performance appears to be disappearing, performance differences that emerge in middle schools and continue throughout high school are in critical areas of mathematics (Hyde, Fennema, & Lamon, 1990; Hyde, Lindberg, Linn, Ellis, & Williams, 2008). One such area is problem solving (NCTM, 2000). To problem solve successfully, it is necessary to have an understanding of underlying concepts. Tasks that require problem solving become more abundant and more difficult beginning in middle school. Engaging in such tasks is essential for learning mathematics and successfully navigating through higher-level mathematics courses (NCTM, 2000).

Research also reveals that the problem solving advantage boys have later in school may result from experiences and problem solving methods used early in school. In particular, gender differences in problem solving strategies begin to appear as early as preschool (Carr & Jessup, 1997; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Moffatt, Anderson, Anderson, & Shapiro, 2009). According to the Fennema and colleagues (1998) study, boys and girls in early elementary school have similar performance outcomes on arithmetic tasks, but more girls than boys used concrete and teacher-instructed strategies such as modeling, counting, and standard algorithms to solve addition and subtraction problems, whereas more boys than girls used invented strategies.

Consistent with previous work, I will define invented strategies as conceptually appropriate strategies that involve mentally breaking down quantities into their place value parts and then recombining the quantities using known number facts and an understanding of place value. For example, a child may solve 28 + 37 by saying "20 + 30 is 50, 8 more is 58, take 2 from the 7 makes 60, and 5 left is 65." These strategies do not require the use of physical materials or counting. Children may invent the strategies themselves or acquire the strategies by observing other children (Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Fennema et al., 1998; Hiebert & Wearne, 1992, 1996). Children that use invented strategies often work from left to right (Fuson, 2003). The term invented is used because these strategies usually are not taught by the teacher. It is important to note that invented strategies necessarily rely on place value knowledge whereas the standard algorithms can be executed without it.

Children who focus on rule following may be unable to adapt their rule on novel problems, which may limit their problem solving experiences and performance. However, using invented strategies can be considered a form of problem solving. Children who use invented strategies might have more experiences solving problems based on conceptual understandings than

children who rely on counting or rule following. Therefore, children who use invented strategies may be more likely to adapt the strategy to solve novel problems. Early experiences solving problems based on understanding underlying concepts may support a child's ability to problem solve later. If this is true, and if boys use invented strategies more than girls in the early grades, then boys might be better prepared than girls for problem solving activities in the later grades.

Two studies will be presented in this paper. Study 1 is a replication of the Fennema and colleagues (1998) study using an existing data set collected about the same time period (Hiebert & Wearne, 1992, 1996). This study will compare gender performance and strategy use by reexamining the strategies first-, second-, and third-grade children used to solve multidigit addition and subtraction story problems that required regrouping.

In a response to the Fennema and colleagues (1998) study, Sowder (1998) suggested a reexamination of existing data on children's addition and subtraction reasoning to test he robustness of the finding. She further stated, "if differences can be replicated, then it would be important to study the reasons for the differences" (Sowder, 1998, p. 13). Following Sowder's recommendation, Study 2 will extend the Fennema and colleague (1998) study by eliciting the reasons children give for the strategies they use.

The following questions guides these studies: (a) How do boys and girls solve multidigit addition and subtraction story problems that require regrouping? Are there gender differences in boys' and girls' correct performance? Are there gender differences in the types of strategies that boys and girls use? and (b) What strategies do boys and girls prefer? What reasons do boys and girls provide for their preferences? Are there gender differences in children's preferences?

Study 1 will address the first set of research questions using an existing data set not previously analyzed for gender differences, while Study 2 will address both sets of research questions using a new data set. The goal for this sequence of studies is to contribute to our understanding of the early origins of gender differences in mathematics and possibly the factors that influence children's use of certain strategies.

Study 1

The purpose of Study 1 is to determine whether boys and girls use and are more successful with different strategies in Grade 1 through Grade 3. To replicate previous research findings, this study examines whether more girls than boys use concrete and teacher-instructed strategies such as modeling, counting, and standard algorithms to solve addition and subtraction problems, and whether more boys than girls use invented strategies.

Methods

Sample. In the Hiebert and Wearne (1992, 1996) three-year longitudinal study, 36 boys and 36 girls were randomly selected to participate in eight individual interviews. The data from the individual interviews is the sample for Study 1. When the students began the study in first grade, 48 were in experimental classes and 24 were in control classes. The experimental classes implemented conceptually based instruction using place value to add and subtract. The control classes followed the classroom teacher's textbook-based approach. As the students moved into second and third grade, many of them moved through both experimental and control classes (see Hiebert & Wearne, 1992, pp. 100-102, for further details).

Interview tasks. The tasks selected for analysis from the individual interviews are the 13 multidigit addition and subtraction story problems that require regrouping. Tasks that require regrouping make it easier to distinguish between invented strategies and standard algorithms.

The story situations chosen for analysis are join with the result unknown (a + b = ?) and separate with the result unknown (c - a = ?). In a join story situation, the initial quantity changes by adding on to it. For example, "Mary has 167 marbles. Thomas gave her 41 more marbles. How many marbles does Mary have now?" In a separate story situation, the initial quantity changes by removing from it. For example, "The Brownies had 574 boxes of cookies. They sold 46 boxes. How many boxes did they have left?"

The participants did not have access to manipulatives (e.g., base-ten blocks, snap cubes, etc.), but had access to a blank sheet of paper and pencil for some of the tasks (Hiebert & Wearne, 1992, 1996). Tasks increased in complexity from interview to interview.

Strategies. Children may use many different strategies to solve the addition and subtraction problems. In general, they progress from modeling or counting strategies to invented strategies and the standard algorithms (Carpenter et al., 1997; Fusion, 2003; Hiebert & Wearne, 1992, 1996). This study uses a strategy categorization similar to the Hiebert and Wearne (1992, 1996) and the Fennema and colleagues (1998) studies.

There are several types of counting strategies. Counting by ones includes counting all, counting up for subtraction, and counting on (from first, last, smallest, or largest) for addition. Counting only by ones may mean that children have not mastered the base-ten structure of the number system (Carpenter et al., 1997).

Strategies that were tens, based and solved mentally from left to right using number facts will be referred to as invented strategies in this paper. In other words, children must demonstrate that they mentally decomposed and recomposed numbers by tens and ones. I will refer to this form of composing and decomposing as understanding place value. There are three main types of invented strategies: sequential, combining units, and compensa-

tion (Carpenter et al., 1997; Hiebert & Wearne, 1992, 1996). Children that use the sequential strategies consider the sum to be a running total. For example, to solve 53 - 29, one can remove 20 leaving 33, then remove 9 by removing 3 to make 30 then 6 to leave 24. Children that use combining-units may individually combine or separate like units. For example, to solve 28 + 35 one can do the following: "20 + 30 is 50, 8 + 5 is 13, 50 + 10 is 60, plus 3 more is 63." Children that use compensation strategies may adjust the numbers to simplify the calculation. For example, "28 + 35 is like 30 + 33, which is 63."

Standard algorithms are efficient procedures for solving arithmetic problems and are often taught to children by teachers or parents (Fennema et al., 1998). However, it is possible to follow the standard algorithm procedures without an understanding of place value. Unlike invented strategies, children that use standard algorithms might not refer to the fact they are combining like units (Carpenter et al., 1997). As defined by Hiebert and Wearne (1992, 1996), any strategy that solves an arithmetic task from right to left and regroups in sequence were considered the standard algorithm.

Results

Performance and strategy scores. The mean and standard deviations for performance scores and strategy scores are reported by gender and grade level, as shown in Table 1 and 2. A performance score is the total number of items with a correct response. These scores were calculated separately for each grade level and by aggregating across Grades 1-3.

There are two types of strategy scores. An "Attempt" strategy score is the total number of items where a specified strategy was used whether or not it

Table 1. Performance and Strategy Scores by Gender and Grade Level

		Mean number of items (SD)						
			Counting		Invented Strategies		Standard Algorithms	
Gender	n[a]	Performance	Attempt	Correct	Attempt	Correct	Attempt	Correct
			Grade 1 (n[b] = 3 items)					
Boys	33	0.58 (0.79)	0.18 (0.39)	0	1.03 (0.88)	0.45 (0.75)	0.12 (0.33)	0.09 (0.29)
Girls	33	0.33 (0.60)	0.21 (0.49)	0	0.73 (0.84)	0.21 (0.55)	0.18 (0.39)	0.09 (0.29)
			Grade 2 (n[b] = 8 items)					
Boys	33	4.27 (1.88)*	0.12 (0.33)	0.06 (0.24)	2.00 (1.85)*	1.55 (1.58)*	2.73 (1.23)	2.61 (1.22)
Girls	31	3.35 (1.66)	0.26 (0.58)	0.10 (0.30)	0.94 (1.12)	0.58 (0.99)	2.94 (1.15)	2.61 (1.20)
d		0.52			0.69	.73		

[a]Number of children in each group (36 boys and 36 girls) who complete each interview. [b]Number of interview items. *d* = effect size *p < .05, two-tailed.

Table 2. Performance and Strategy Scores by Gender and Grade Level - *Continued*

		Mean number of items (SD)						
			Counting		Invented Strategies		Standard Algorithms	
Gender	n[a]	Performance	Attempt	Correct	Attempt	Correct	Attempt	Correct
			Grade 3 (n[b] = 2 items)					
Boys	30	1.28 (0.80)	0	0	0.17 (0.38)	0.13 (0.34)	1.17 (0.75)	1.13 (0.78)
Girls	28	1.57 (0.57)	0.11 (0.32)	0.04 (0.19)	0.11 (0.32)	0.07 (0.26)	1.54 (0.58)	1.46 (0.58)
			Grade 1 – 3 (n[b] = 13 items)					
Boys	28	6.10 (3.0)	0.32 (0.48)	0.07 (0.26)	3.32 (2.65)*	2.29 (2.35)*	3.93 (1.62)	3.76 (1.71)
Girls	26	5.35 (2.35)	0.65 (0.94)	0.08 (0.27)	1.78 (1.91)	0.92 (1.62)	4.63 (1.67)	4.27 (1.69)
d					0.67	0.68		

[a]Number of children in each group (36 boys and 36 girls) who complete each interview. [b]Number of interview items. *d* = effect size *p < .05, two-tailed.

led to a correct solution. The "Correct" strategy score is the total number of items with a correct solution for a specified strategy. Strategy scores were calculated separately for each grade level and by aggregating across Grades 1-3.

A two-tailed *t* test was used to compare boys' and girls' performance and strategy use. Cohen's *d* effect size with pooled standard deviations was reported when significant differences between boys and girls were found, with the higher mean starred. Small, medium, and large significant differences exist with an effect size of 0.2, 0.5, and 0.8 respectively.

There were no significant differences between boys' and girls' correct performance in Grade 1, Grade 3, and Grades 1-3 combined. In Grade 2, boys obtained significantly more correct solutions than girls did with an effect size of 0.52. These results differ slightly from the Fennema and colleagues (1998) findings which did not find performance differences in first, second, or third grade.

Results from the children's strategy scores are also reported in Table 1 and 2. The mean and standard deviation for Attempt and Correct strategy scores are reported by gender and grade level. This study found no differences for counting strategies for any grade level. For invented strategies, second-grade boys attempted significantly more invented strategies than did second-grade girls with an effect size of 0.69. Second-grade boys also obtained significantly more correct solutions than girls when using invented strategies with an effect size of 0.73. The results differ slightly from the Fennema and colleagues (1998) study where second- and third-grade girls tended to use counting strategies and boys tended to use invented strategies.

In Grades 1-3 combined, boys attempted significantly more invented strategies than girls with an effect size of 0.67. Boys also obtained signifi-

cantly more correct solutions than girls when using invented strategies with an effect size of 0.68. It is important to note than in this study there were fewer items in first and third grade than in second grade. This may account for the significant difference that appeared only in second grade and for the significant difference when the three grades were combined.

Strategy group trends. A descriptive analysis explored possible differences in the sequence of children's strategy use. This analysis may reveal gender differences in strategy use which are not apparent in the previous analysis that combines data across several interviews. Five groups of strategy trend categories were defined. Over the three-year period, the children obtained correct answers for multidigit addition and subtraction story problems by (a) only using standard algorithms by the end of second grade (13 girls, 10 boys), (b) using counting strategies for one addition problem during the December interview in second grade, then shifting strictly to standard algorithms by the end of second grade (2 girls, 1 boy), (c) using invented strategies then shifting strictly to standard algorithms by the end of second grade (6 girls, 12 boys), (d) alternating between invented strategies and standard algorithms (3 girls, 4 boys), and (e) using counting, invented strategies, and standard algorithms without a detectable pattern (2 girls, 1 boy).

Based on the strategy trends that occurred over the three years, two groups of students were identified. The Invented Strategies group consisted of 17 boys and 11 girls. Similar to the invented algorithm group in the Fennema and colleagues (1998) study, the Invented Strategies group used

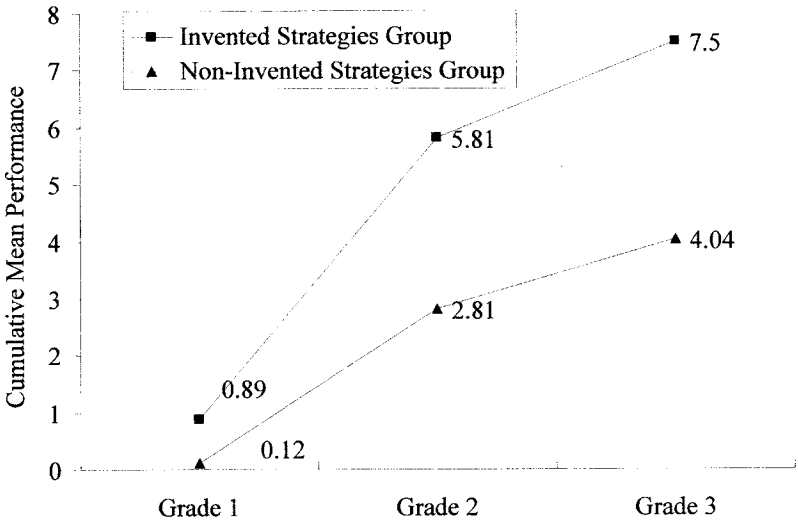

Figure 1. Cumulative mean performance in Study 1 as a function of grade level and strategy groups.

invented strategies to obtain a correct solution for at least one task. The Non-Invented Strategies group consisted of 11 boys and 15 girls. Similar to the standard algorithm group in the Fennema and colleagues (1998) study, the Non-Invented Strategies group did not obtain a correct solution using invented strategies for any multidigit addition or subtraction story problem. A majority of their correct solutions were obtained by using the standard algorithms.

The performance scores of the Invented Strategies Group and Non-Invented Strategies groups were compared to determine whether children who used invented strategies have an advantage over children who do not. Figure 1 shows the cumulative mean performance at the end of each grade level for the Invented Strategies and Non-Invented Strategies groups. Over the three years, the Invented Strategies group correctly solved significantly more multidigit addition and subtraction story problems requiring regrouping than the Non-Invented Strategies group, with $p < 0.0001$ and an effect size of 1.63. The results are similar to the Fennema and colleagues (1998) findings where the invented algorithm group outperformed the standard algorithm group on extension problems (three-digit numbers) given in third grade.

The Invented Strategies group might be developing an early advantage in problem solving because they are using their understanding of the underlying place value concepts to create appropriate solution strategies. This advantage appears to favor boys even though the unequal distribution boys and girls within the Invented Strategies and Non-Invented Strategies groups is not statistically significant ($p = 0.18$).

Discussion

This study replicates some, but not all, of the Fennema and colleagues (1998) findings. First, unlike the Fennema and colleagues (1998) study, performance differences were found in second grade where boys answered significantly more questions correct than girls did. Second, Fennema and colleagues (1998) found that girls were more likely to use modeling or counting strategies in second and third grade. Due to the design of this study, boys and girls infrequently used counting strategies and gender differences in counting strategy use did not exist in any grade level. Finally, Fennema and colleagues (1998) found that girls used the standard algorithm significantly more than boys by the end of third grade. Although not significant, this study had a similar trend.

Overall, the findings suggest that gender differences in invented strategy use exist. Although the size of the differences is smaller than those reported by Fennema and colleagues (1998), the significance of these results can be found in the collapsed group analyses. Children who used invented strategies performed significantly higher than children who did not. Even though gender differences within the groups were not significant, the trends are

consistent with Fennema and colleagues (1998) report that boys were more likely to use invented strategies and girls were more likely to use standard algorithms.

Study 2

To explore the reasons for children's strategy use, Study 2 was designed to elicit children's preferences in strategy use. Children were prompted to use different strategies on consecutive addition or consecutive subtraction problems that were similar. They were then asked to identify their favorite addition (subtraction) strategy and to provide a reason for choosing that method. The goal of this study was to gain new insights into the sources of gender differences in strategy use.

Since research indicates that girls adopt the values of an authority figure more quickly than boys (Fagot, 1978; Fennema, 1985; Gold, Crombie, Bender, & Mate, 1984; Maccoby & Jacklin, 1974), children's strategy preferences may reveal that boys and girls are influenced by an authority figure when understanding what it means to learn and do mathematics in different ways. In other words, I hypothesized that if a child chose a teacher-taught strategy (i.e., the standard algorithm) as their favorite strategy, they would emphasize that the strategy was taught to them by an authority figure. In contrast, if a child chose invented strategies as their favorite strategy, they would emphasize sense making in lieu of following the directions of an authority figure.

Method

Sample. Students from one elementary school in Delaware were chosen to participate in Study 2. The sample consisted of 70 third graders (33 boys and 37 girls) representing a range of socioeconomic levels and various ethnic groups.

The students attending this elementary school were chosen to participate in the study because the second-grade teachers used the Investigations in Number, Data, and Space textbook series (Economopoulos & Russell, 1997) a standards-based mathematics curriculum that provides students an opportunity to invent strategies to solve addition and subtraction problems before being introduced to the standard algorithms. The textbook series includes six to seven weeks of addition and subtraction instruction using sums up to 100 by exploring 100's charts, parts and wholes, and problems with missing parts (Economopoulos & Russell, 1997). According to the textbook series description, "Students will not need to learn the historically taught algorithm that uses carrying and borrowing because they will develop their own algorithms based on sound understanding of the structure of numbers and operations" (Economopoulos & Russell, 1997, p. 50). The second-

grade curriculum is important because the third grade student interviews occurred at the beginning of the school year before the addition and subtraction instruction. Thus, the participant were most likely using the skills they developed by the end of second grade.

Interviews. Each child participated in one individual interview for a maximum of one hour conducted by the researcher. All participants had access to paper with markers (to prevent erasing), unlinked snap cubes, and craft sticks with rubber bands. Demonstrations of making groups of ten were not provided. Story problems were presented to the children on a single sheet of standard size paper and read in its entirety as often as necessary. Further story problem details are presented in the next section.

During all interviews the same follow-up questions were asked to elicit students' explanations about the strategies they used and the strategies they preferred to use. A sample of questions include "Can you tell me how you figured that out?", "What numbers did you use to get that answer?", "You solved this set of problems using different strategies (pointing to different strategies), which is your favorite way?", "Why do you like counting with blocks better than solving the problem on paper?", and so on. Further questions depended on the student's responses. The students' thinking was probed until the response clarified the strategy, and/or the reason for their strategy preference, or until it was obvious that the students would not provide further explanation.

A coding rubric was used for each student during the interviews to record visible strategies and students' explanations. All interviews were audio taped to test coding reliability. When needed, audiotapes were also used to complete the students' explanations and the description of their strategies. Interviews were completed before the third grade multidigit addition and subtraction lessons were conducted.

Interview tasks. The interview tasks whose results were used for data analysis included three join-result unknown and three separate-result unknown story problems that require regrouping. Two consecutive addition and two consecutive subtraction problem requiring regrouping of two-digit numbers (28 + 37 = ?, 47 + 26 = ?, 63 - 27 = ?, and 74 - 36 =?) were included in the interviews. Participants had multiple experiences solving problems of this type in their classrooms by the time the interview was conducted.

Participants in a pilot study at the end of second grade seemed unfamiliar with adding and subtracting three-digit numbers, and subtracting across zero. Therefore, on addition problem (167 + 41 = ?) and one subtraction problem (90 - 49 = ?) were considered novel problems. The addition problem required regrouping only in the tens place, and the subtraction problem required regrouping with the zero in the ones place. These problems were used to assess the children's ability to extend their strategies to solve unfamiliar problems.

Children were asked to solve each story problem using any strategy they wished except for one addition problem and one subtraction problem. On these problems, children were asked to use a different strategy than the one used on a previous but similar problem. For example, on the consecutive addition problems, if a child solved the first problem (28 + 37 = ?) using manipulatives, then he/she was not permitted to solve the second problem (47 + 26 = ?) using manipulatives. The child could use a mental strategy or work the problem on paper. This helped determine whether or not children were capable of using multiple strategies and provided the prompts used to assess preferences. After attempting to use an alternate strategy, the children were asked to think about which strategy they preferred to use and why.

Strategies. All of the strategy descriptions from Study 1 apply to Study 2. In addition, children in Study 2 used modeling strategies by representing the problem situation using manipulatives or by drawing figures on paper. To find a solution using these strategies, children may count all of the objects to find the sum or count all of the remaining objects to find the difference. They may count the objects by ones, twos, fives, or tens. From hence forth, modeling strategies will also be considered counting strategies for coding purposes.

Children that model the story situation using tens material make groups of ten with the objects and count by ten. For addition problems, they may regroup the ones to make a group of ten, which is similar to carrying. For subtraction problems, they may ungroup a ten to make ten ones, which is similar to borrowing. Although modeling with tens material shows some place value understanding, it is considered a counting strategy because objects were needed to keep track of quantities. In strategies classified as invented strategies, children were able to mentally manipulate tens and ones. Therefore, counting by tens using materials are classified as counting strategies and count by tens without the use of materials will continue to be classified as invented strategies.

Strategy, preference, and reason codes reliability. To test for coding reliability, 10% (7) of the participants (6 items each) were randomly selected and were coded by the author and an assistant. Coders independently selected strategy use codes for each item for each child while reviewing the audiotapes and the child's handwritten notes. Forty of the 42 coded items received the same strategy code for 95.2% code reliability.

Strategy preferences are the strategies children say are their favorite strategies. Strategy preference reasons are explanations children give for choosing a strategy to be their favorite strategy. The strategy preference reasons were transcribed from the audio-taped interviews and corresponding handwritten notes. Categories were created for the strategy preference reasons by reviewing the children's responses.

Again, responses from 10% (7) of the participants were randomly select-

ed. The preference and reasons were coded independently by both coders. Intercoder reliability was 100% for the strategy preferences. Thirteen of the 14 strategy preference reasons received the same code for 92.9% reliability.

Results

Performance and strategy scores. The mean and standard deviation for performance scores and strategy scores are reported by gender in Table 3. Effect size is reported when significant differences between boys and girls were found. The higher mean is starred. Similar to Study 1, no significant correct performance difference was found between boys and girls over all items.

Table 3. Third Grade Performance and Strategy Scores by Gender

			Mean number of items (SD)						
				Counting		Invented Strategies		Standard Algorithms	
n[a]	Gender	n[b]	Performance	Attempt	Correct	Attempt	Correct	Attempt	Correct
6	Boys	33	2.91 (1.74)	1.91 (1.81)	0.97 (1.16)	2.09 (2.44)*	1.58 (1.94)*	1.39 (1.62)	0.64 (1.17)
	Girls	37	2.24 (1.44)	3.11 (1.76)*	1.27 (1.07)	0.49 (1.35)	0.14 (0.48)	2.24 (1.98)	1.03 (1.24)
	d			0.67		0.81	1.02		

[a]Number of interview items. [b]Number of children in each group who complete the interview. *d* = effect size *p < .05, two-tailed.

In general, girls attempted significantly more counting strategies than boys with an effect size of 0.67. Although not significantly different, girls had more correct solutions when using counting strategies than boys. Boys attempted to use invented strategies significantly more than girls with an effect size of 0.81. Boys also obtained significantly more correct solutions using invented strategies than girls with an effect size of 1.02. Study 2 confirms the third-grade finding for the Fennema and colleagues (1998) study that was not confirmed in Study 1. In fact, the effect sizes are as large as those reported by Fennema and colleagues (1998). Although not significant, the trends suggest that girls attempted to use the standard algorithms more often than boys and got more correct solutions than boys when using the standard algorithms. The direction of the difference is same as that reported in the Fennema and colleagues (1998) study.

Strategy groups with performance patterns. Children were separated into one of four groups based on their correct responses for addition and subtraction problems.

- Multiple Strategies Group: 8 boys and 3 girls successfully used more than one strategy, which includes invented strategies, with a mean performance of 4.36 (0.81 SD).
- Invented Strategies Only Group: 7 boys and no girls were only successful solving problems using invented strategies, with a mean performance of 3.86 (1.22 SD).

- Counting and Standard Algorithm Group: 5 boys and 16 girls successfully used counting strategies and standard algorithms but not invented strategies, with a mean performance of 2.9 (1.26 SD). These children either had not success with inverted strategies or did not attempt to use invented strategies.
- Counting or Standard Algorithm Group: 13 boys and 18 girls were only successful using counting strategies or only successful using the standard algorithms with a mean performance of 1.39 (1.15 SD). These children either had no success with invented strategies or did not attempt to use invented strategies.

A one-way ANOVA test was used to determine whether there were significant differences between the performance scores of the groups. Compared to the other groups, the sample size of the Invented Strategies Only group was too small to be included in the analysis as a separate group. Therefore, the Invented Strategies group and Multiple Strategies groups were combined to form a new Invented Strategies group, which obtained correct solutions through the use of invented strategies. The Invented Strategies group is composed of 15 boys and 3 girls with a mean performance of 4.17 (0.99 SD).

Results from the ANOVA showed significant performance differences between the three groups with $p < 0.0001$. In particular, the Invented Strategies group outperformed the Counting and Standard Algorithm group with $p < 0.001$ and effect size of 1.12. The Invented Strategies group also outperformed the Counting or Standard Algorithm group with $p < 0.0001$ and effect size of 2.60. The Counting and Standard Algorithm group outperformed the Counting or Standard Algorithm group with $p < 0.0001$ and effect size of 1.25. It appears children benefit from successfully using multiple strategies and they benefit most when this includes using invented strategies.

No performance differences were found between boys and girls within each group. Rather, the gender differences become visible when considering the number of boys and girls distribution among the three groups were not the same ($p < 0.0001$).

To compare these results with Study 1 and the Fennema and colleagues (1998) study, the Counting and Standard Algorithm group and the Counting or Standard Algorithm group were combined to form the Non-Invented Strategies group. Children in the Non-Invented Strategies group did not use invented strategies to obtain a correct solution. The Non-Invented Strategies group is composed of 18 boys and 34 girls with a mean performance of 2.0 (1.4 SD). Children that used invented strategies performed significantly higher than children that did not use invented strategies with $p < 0.0001$ and effect size 1.79. A Chi-Square analysis shows that the distribution of boys and girls in the two groups were not the same ($p < 0.0001$). Boys were equally likely to be in the Invented Strategies group or Non-Invented Strategies group. Girls were not likely to be in the Invented Strategies group.

These results are similar to the findings from the Study 1 and the Fennema and colleagues (1998) study.

Strategy preferences. Strategy preferences are the strategies children say are their favorite strategies. Chi-square analysis shows that boys and girls differed in their choices for favorite addition ($p < 0.02$) and subtraction ($p < 0.002$) strategies. Examination of the strategy preference revealed that girls were most likely to choose counting strategies and standard algorithms and unlikely to choose invented strategies as their favorite strategies. Boys appeared to be equally like to choose any strategy as their favorite strategy. Children's preferred strategies are consistent with the types of strategies they use to solve addition and subtraction problems.

Reasons for preferences. What reasons did children give for using particular strategies? Children's strategy preference data were treated qualitatively due to the limited amount of data available in each category. The three major mathematics-related reasons expressed for the three strategy types are accuracy, speed, and ease of use (see Table 4). Strategy preference reasons for addition and subtraction strategies have been combined in Table 4 because the reasons were similar across both types of problems.

Table 4. Children's Strategy Preference Reasons by Percentage of Children and Gender

	Counting		Invented Strategies		Standard Algorithms		Total
Preference Reasons	Boys	Girls	Boys	Girls	Boys	Girls	
Ease of Use	6	6	17	0	31	22	15
Speed	6	6	39	50	15	33	20
Accuracy	59	67	5	50	15	22	38
Makes sense	0	12	28	0	8	0	9
Authority figure	0	3	0	0	0	15	4
Non-mathematical	29	6	11	0	31	8	14
n	17	33	18	2	13	27	110

Note: All children (33 boys and 37 girls) had two opportunities during the interview to identify their strategy preference reasons.

The most common reason for choosing a strategy as a favorite strategy was accuracy. Children who described a strategy as accurate wanted to avoid a miscount that can occur by losing track of quantities while counting. The following quotes are examples of an 'accuracy' preference reason:

Boy 1: For subtraction I like to use the objects [modeling with tens

counting]...it's easier because when you're having to do subtraction you can get mixed up a lot because you can forget the numbers sometimes in your head [when you do an invented strategy]...[but] with objects you can count them to make it easier to remember so you won't lose track.

Girl 1: I like to use craft sticks [counting by ones]. Sometimes I lose track of the numbers so I use craft sticks instead...but, you can lose track with craft sticks too.

Boy 2: [With the standard algorithm] you don't get lost with the counting. The numbers are smaller to work with.

All three children associated different strategies with accuracy. However, all three quotes stress the importance of keeping track while counting when using their favorite strategy or that other strategies make it difficult to keep track while counting.

Children that describe a strategy according to speed seem to refer to the amount of time consumed during the solution process. Speed was the second most common strategy preference reason. It appears that time consumption seemed to be a concern for many children in Study 2. The following quotes are examples of a 'speed' preference reason:

Boy 3: [I like to solve problems in] my head [using an invented strategy] because I can just think things really fast that's why I didn't use a piece of paper because I can do things really fast in my head and it doesn't take long.

Girl 2: Sometimes I use different things but usually I do it [an invented strategy] in my head...it's just easier...because we're timed in class sometimes and it takes a long time to count out if you had like a lot of blocks it takes a long time to do that.

Boy 4: [The standard addition algorithm is] a faster way to answer the questions.

Girl 3: [I like the standard addition algorithm] because it's easier because see how long it took to take out all the cubes. That's why I like on the paper better because it doesn't take as long.

These quotes show that regardless of gender, boys and girls value speed for

the same types of strategies. Girls chose invented strategies as their favorite strategy for the same reason as boys do. Similarly, boys chose the standard algorithm as their favorite strategy for the same reasons as girls do. Speed appears to be a highly valued mathematical skill when solving problem using invented strategies or the standard algorithm.

The third most common preference reason was 'ease of use.' Children that describe a strategy as easy to use seem to refer to the degree of difficulty they experience when executing the solution process. Therefore, ease of use, speed, and accuracy were treated as separate preference reasons. Children who referred to ease of use appeared to be concerned with the process of obtaining a solution. Children who referred to speed appeared to be concerned about obtaining a solution quickly. Children who referred to accuracy appeared to be concerned about obtaining the correct solution. The following quotes are examples of an 'ease of use' preference reasons:

Boy 5: Writing [the standard addition algorithm is my favorite way to solve the problems] because it was the easiest thing. You only have to write what the two questions were and solve it. [I didn't choose counting with Craft sticks] because that would be too [physically] difficult...because I kept on putting the rubber band on them and then I had to take them off.

Girl 4: Writing them down [using the standard addition algorithm is my favorite strategy] because when I write them down, I like find the answer easier than when getting these [snap cubes] and taking them out.

Girl 5: [I like to solve it on] paper [using the standard addition algorithm] because it's easy because you're just counting this row [one's column] to get this answer and then the other row [ten's column] to get that answer and then put the other one on top this row [ten's column]. And then you get your answer.

Girl 4 and Boy 5 compared their favorite strategy to other strategies. They describe their favorite strategy as being easy to use because it is less physically or mentally demanding than other strategies. Girl 5 does not compare her strategy to other strategies. Instead, she describes the simplicity of the standard algorithm. Girl 5 views the standard algorithm as two separate problems that have smaller quantities to count. She believes it is easier to count smaller quantities when solving the problem in parts, then larger quantities when solving the problem as a whole.

The rarest strategy preference reasons were sense making or the influence of an authority figure. Children that associated their favorite strategy with sense making seemed to refer to knowing or understanding a solution process. The following quotes are examples of a 'sense making' preference reason:

Boy 6: Well, if I'm using higher numbers [I like to solve them using an invented strategy by their parts: (10s and 1s] because it helps me to do it instead of trying to do the whole thing at once [such as counting the entire number by ones]...I learned [the standard algorithm] in second grade...but I lost it over the summer.

Girl 6: Well, [when] taking away [my favorite strategy] is with these [counting with snap cubes] because it's easy...Adding is easy on paper [using the standard addition algorithm] but subtracting like this [with the standard subtraction algorithm], I just don't see [how to do it]. For adding [I can see how to use the standard addition algorithm, but], I just don't see how to subtract these [numbers using the standard subtraction algorithm], but for subtracting if I take 90 [snap cubes] and I'm taking away 49 [snap cubes] it's easy because I'm taking away when I'm counting.

Boy 7: [I like invented strategies] because I work with numbers I know [sum and differences] then add the other numbers in.

These children found the standard algorithm process confusing. Girl 6 chose to use a counting strategy because it logically represents the meaning of subtraction. Boy 6 chose an invented strategy and demonstrated his understanding of place value by combining like units. Boy 7 described breaking number down into known number facts to find his solutions when using invented strategies. In each case, these children preferred strategies whose solution process made sense.

Even though sense making was a preference reason most associated with invented strategies, very few children chose sense making as a strategy preference reason. Therefore, there is no strong evidence that boys (or girls) prefer invented strategies because the strategies make sense.

Children that referenced an authority figure identified the authority figure as their resource for learning the strategy or using the strategy. The following quotes are examples of an 'authority figure' preference reason:

Girl 7: [I] like counting by tens with craft sticks because I'm use to it. I did it with my Dad over the summer.

Girl 8: I like to do [the standard algorithm] because it's easier. Because my Dad taught me if you put the basic numbers and you add up the ones first and if they are higher than 9 and it's a double digit number and you put that one there [under one's column] and you put the other number in the other column basically the number in the one's column goes right there and then you count the other two numbers and you would put it over there.

Girl 9: [My favorite way to solve the problems is with the standard algorithm by] paper because my teacher said when you use subtraction you can use the higher number first then you take it lower. That makes it easier for me.

Only 5 children (all girls) express that they chose a strategy as their favorite strategy because they learned it from an authority figure such as a parent or teacher. All of these girls did not attempt invented strategies to solve the addition or subtraction problems but used more than one strategy to obtain correct solutions. Although these preference reasons are consistent with a hypothesis that girls attend more than boys to the teacher's directions, there is insufficient strategy preference data to confirm this hypothesis.

Table 4 also shows that some reasons seem more associated with some strategies than other. For example, children that preferred counting strategies most often identified accuracy as their reason for preferring counting strategies dues to the ease of tracking large quantities without losing count. The preference for counting strategies appears to be a valuing of visualizing or physically touching objects to accurately track and remember the quantities they need to add or subtract.

Children that preferred invented strategies most often found them to be less time consuming than other strategies, particularly counting strategies using manipulatives. The next most common reason for preferring inventing strategies was sense making. Although sense making was not frequently reported, all the children that preferred invented strategies demonstrated their knowledge of place value when they were required to explain their solution process.

Children that preferred the standard algorithm equally expressed that these strategies were fast, easy to use, and often lead to correct solutions. In general, the preference for the standard algorithms appeared to be a rejection of the more mentally taxing invented strategies or inefficient counting strategies for large numbers. For example:

Boy 10: [My favorite way to solve subtraction problems is] on paper [using the standard subtraction algorithm] because you

can write them out...because you can't really subtract with those [craft sticks] because it's harder...well because if you go like 'this one craft stick equals up to ten' that's kind of easier but if you take 90 craft sticks then that's pretty hard...I don't know [what makes the standard subtraction algorithm better]. You don't get lost with the counting. The numbers are smaller to work with."

Girl 12: [I] like that [pointing to the standard subtraction algorithm] because I like borrowing from second grade. It was really fun...and I like the pictures also, I like drawing [tally marks] so I can't tell from this one or the paper...[But with tally marks] you can make mistakes when drawing the lines, you might be probably doing it too fast and might not count one by mistake and get the wrong answer sometimes. But this one [the standard algorithm]...it would be easier to get the number for this one because you can get the one answer for each of those [pointing to tens and ones columns].

Girl 13: I like doing it on paper best [using the standard addition algorithm] because it's really slow when you're fixing these [craft sticks] because I just forget how to count with these. [The standard addition algorithm is better] because I can actually count with it...if you put a line, like an imaginary line down [between the tens and ones column] you do the back ones first then the front ones and that's easier for me.

These children expressed the difficulty they experienced when keeping track of large quantities mentally or with manipulatives. They chose the standard algorithm as their favorite strategy because they believed it was a fast and easy to use strategy. They described performing operations on the numerals in the ones column and tens column as smaller and separate ones-count problems. How these children describe the standard algorithm appears to reflect their lack of understanding of how place value is represented in the solution process. Unfortunately, many children chose to use standard algorithms even though they do not understand why the process worked. The strategy preference reasons, strategy use, and performance during the individual interview suggests that these children may not have a full understanding of place value concepts before beginning to learn the standard algorithms for addition and subtraction. It may also explain the diverse performance on the addition tasks, and the poor performance on the subtraction tasks.

In summary, children's strategy preference reasons did not vary great-

ly. Counting strategies, invented strategies, and the standard addition and subtraction algorithms were all associated with accuracy, speed, and ease of use. However, the degree to which these labels fit each strategy varied greatly. In particular, these labels were defined when strategies were compared to each other and seemed to depend upon the degree to which children understood place value concepts.

Discussion

Gender differences in strategy use that appeared in the Fennema and colleagues (1998) study were replicated to some extend in Study 2. Over all the tasks, performance differences were not found. However, gender differences in strategy use were present. Although there were no differences between the success rates of boys and girls when using counting strategies or the standard algorithm, girls used them more often than boys. In contrast, boys used invented strategies more than girls, and boys obtained more correct solutions using invented strategies than girls.

Also similar to the Fennema and colleagues (1998) study and Study 1 are the group analyses which underscore the significance of the previous findings. Children that used invented strategies performed significantly higher than children that did not. Boys and girls also had similar performance scores when using invented strategies, which suggests that the use of invented strategies matters more than gender.

Children's strategy preferences provide some insight into the kinds of strategies children liked to use to solve addition and subtraction problems and why those preferences might reinforce gender differences in strategies use. Girls were more likely to choose counting strategies and the standard algorithms than invented strategies as their favorite strategies. Boys were equally likely to choose any strategy as their favorite strategy but more likely than girls to choose invented strategies as their favorite strategy. Boys' and girls' reasons for choosing a strategy as their favorite strategy did not vary greatly but a small number of girls cited their teacher or parent as the major source. Children's strategy preference appear to be related to their understanding of place value concepts or the degree to which they understood or remembered the standard algorithm procedures.

In general, most of the children that chose ones-count strategies as their favorite strategy felt that the simplicity of counting items one at a time was an easy way to add and subtract quantities. They relied on being able to visualize the items to reduce the stress of tracking the items and remembering what needed to be counted.

For those children that actually counted by tens, the process appeared to the observer to require more time than counting by ones. However, it appeared that these children were also trying to make sense of the process as if they were in the beginning stages of developing understanding of place

value. Perhaps children who were developing their understanding of place value preferred to work with the tens count strategy because it made sense rather than the standard addition and subtraction algorithm which may have been more confusing than helpful or efficient to them. Perhaps children with more experience and a better understanding of place value chose invented strategies when it became easier to keep track of the numbers in their heads.

The standard algorithm for addition and subtraction was also considered a fast strategy. However, it was very seldom that children displayed an understanding of place value when using the standard algorithms. Only seven children verbally differentiated between tens and ones while regrouping. Four of these children solved other problems using invented strategies. Often children preferred speed when they considered the alternative of counting strategies by ones. They referred to the fact that using the standard algorithm was faster and easier to handle the smaller quantities in each column. For these children, it often appeared to the observer that the speed of calculating the mathematics was more valuable than understanding the mathematics, particularly since only one out of 40 standard algorithm preference reasons was related to sense making. More specifically, it can be less stressful to recall the rules for a standard algorithm that has the potential of providing a correct answer than to use a more time consuming method.

There were five documented cases where children initially attempted to use the standard algorithm to quickly solve a problem, but abandoned it when they could not recall or make sense of the rules for regrouping. Instead, they resorted back to ones-counting as if realizing that their only choice was to solve the problem the long way. For example, upon asking one girl why she did not complete her solution to 47 + 26 using the standard algorithm she initially attempted, she stated "sometimes I get mixed up so if that happens, I do it a different way."

Conclusions

Findings from Study 1 and Study 2 contribute to the empirical database on early gender differences in problem solving strategies and provide new information pertaining to children's preferences and their reasons for choosing particular strategies. Gender differences on the use of invented strategies exist in favor of boys. Children that used invented strategies performed higher than those who did not and more boys than girls use invented strategies. Although all children valued speed, accuracy, and the ease of using a strategy, children that did not prefer invented strategies rarely demonstrated an understanding of place value concepts.

Based on these children's strategy preferences and strategy use, they appear to develop different values and understandings of what it means to

learn and do mathematics. Particularly, most children believed that producing a correct solution easily or quickly is the most valuable mathematics skill to develop. However, children that preferred counting strategies or the standard addition and subtraction algorithms seemed less likely to value developing their understanding of place value and its relationship to their solution process. On the other hand, most children that prefer invented strategies appear to value understanding place and its relationship to their solution process because they believed it would likely lead to correct solutions in an efficient manner.

These findings suggest that gender differences in mathematics achievement in the later grades favoring boys may originate with gender differences in strategy use in the early grades. Children that used invented strategies are more likely to have advanced levels of understanding than children that use counting strategies or standard algorithms. These findings appear to be consistent with hypotheses from other studies which suggest that gender difference in mathematics may originate before children begin formal schooling (Carr & Jessup, 1997; Fennama, 1998; Moffatt, Anderson, Anderson, & Shapiro et al., 2009). In other words, some children may be building a strong foundation for developing the skills necessary to make sense of more complex mathematical ideas in the future.

Some researchers might interpret the findings with a deficit view of girls where girls need to change or improve their understandings and performance to develop the same kinds of competencies that boys are more inclined to develop. However, there are other ways to interpret these data. As (Boaler, 2002) suggests,

> An important responsibility of gender researchers in the future will be to build on our predecessors' work and search for explanations of the differences they found, not within the nature of girls but within the interactions that produce gendered responses. (p. 139)

In Study 1 and Study 2, girls performed as well as boys. The question is not about capability but about reasons for strategy choices. A plausible hypothesis is that children are influenced by their environment to choose and prefer particular strategies. Although it is difficult to assess the environmental influences of children's strategy use and strategy preference in these studies, children's rationale for addition and subtraction strategy preferences suggest that environmental factors may contribute to gender differences in strategy use. Girls and boys might have varying interpretations of environmental messages that elicit different ways of knowing mathematics.

The evidence indicates that invented strategies are one of the most effective strategies for both boys and girls at this point in their mathematics development. In addition, children benefit most when they are flexible enough

to use multiple strategies as long as invented strategies are included in their repertoire. Invented strategies are the most consistent in producing correct answers and require the most explicit understanding of place value. When girls choose to use these strategies, they are as successful as boys. But the evidence shows that girls choose to use them less often. Why is that?

Children's reasons for choosing a strategy were most often related to accuracy, speed, and ease of use. Why do they appear to believe that speed and accuracy are more important than making sense of place value concepts? Perhaps school instruction and home interactions convey this message through the proportion of time devoted to practice and the way in which assessments are conducted. Could it be that girls are more receptive of this message than boys? Although the data in this study did not confirm this finding from other studies (Fagot, 1978; Fennema & Peterson, 1985; Gold et al., 1984; Maccoby & Jacklin, 1974), this hypothesis is too complex to confirm or deny in a single study. What was apparent from the data reported here is that few children, either girls or boys, expressed explicit value of making sense of mathematics. If girls (and boys) are to use invented strategies as part of their repertoire to solve problems, this value surely needs to be encouraged and made more public in classrooms.

Based on these conjectures, future research on children's strategy use and strategy preferences on arithmetic problems should include an environmental influence component and a more probing measure of the reasons for children's strategy choices. The slight variations in the findings from study to study suggest that the environment has an effect on students' strategy use and possibly their perceptions of the strategies they are expected to use. It is important to learn how conditions of the instructional environment influence gender differences in mathematics, especially because this environment surely affects the values children internalize and, in turn, the solution strategies they choose to use.

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A Change in Questioning Tactics: Prompting Student Autonomy

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Abstract

In this study, we examined the types and the frequencies of questions and explanations exchanged between four teachers and their students in fourth-grade mathematics classes, across two years. Specifically, we observed these classes for one week in two separate, consecutive years; in the first year, they all employed a traditional curriculum, while in year two, they all used a reform-oriented curriculum, *Math Trailblazers.* We examined the discourse practices of these teachers across the two years and how these discourse practices granted the intellectual space for the children to recognize and utilize more autonomy in classroom discussions. We not only offer a quantitative analysis of the changes in classroom discourse practices across the two years, we have also included selected examples of verbal exchanges between teachers and students to showcase notable qualitative differences we observed from year one to year two. Given our findings, we recommend that teachers focus foremost on their discourse practices when attempting to adopt a reform-oriented curriculum.

Traditionally, in U.S. elementary school mathematics classes, a teacher would present a problem, given an algorithm for the solution, and then ask a series of questions intended to insure that students know how to properly apply the algorithm, often providing the answer just to make sure that the students knew what the teacher wanted (Balacheff, 1990; Cazden, 1988; Martin, Hunt, & Lannin, 2001; Mehan, 1979). For many teachers and schools, this pedagogical approach was reasonably effective. The support for this belief was that most students could pass in-class tests and some students could memorize formulas so proficiently that they would solve problems without writing the procedure. As is true for many traditions that produce some positive results, many educators were reluctant to change. However,

in recent years, reform efforts in mathematics have advanced many suggestions about teacher development, student learning, and improvement in U.S. academic performance. These suggestions have served as a major impetus for researchers and educators to reevaluate how mathematics is taught in U.S. schools.

One of the most extensive suggestions for change was brought on by the emergence of the National Council of Teachers of Mathematics' (NCTM, 1989) *Curriculum and Evaluation Standards for School Mathematics,* the NCTM's (1991) *Professional Standards for the Teaching of Mathematics,* and the NCTM's (2000) *Principles and Standards for School Mathematics.* According to the *Standards* (NCTM, 1991, 2000), instead of relying on students to commit formulas to memory, teachers should emphasize problem solving and reasoning, encourage students to make connections between mathematical topics, promote communication of mathematical ideas between students through discussion, and provide opportunities for students to learn procedurally and conceptually rich mathematics. Educators have agreed and repeatedly argued that acquiring a solid and deep understanding of mathematics involves more than learning concepts, principles, and their structure (Hiebert & Carpenter, 1992; Schoenfeld, 1992). "Complete understanding, they argue, includes the capacity to engage in the process of mathematical thinking...framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on" (Stein, Grover, & Henningsen, 1996, p. 456).

Despite this call for change, many teachers find reform efforts difficult. Should these suggestions be implemented in a certain order? Are they dependent on one another? Do you practice all of the suggestions simultaneously? Often, teachers are left with some confusion as to the implementation of reform suggestions and reform curricula goals. This study explores the initial transformation in discourse practices that occurs when teachers adopt a reform-oriented curriculum. More specifically, we examine how four teachers, during this transition, changed their discourse practices to allow students the opportunity to engage in the material in new, different, and non-routine ways.

The *Standards* state, "Today, many students are not learning the mathematics they need. In some instances, 'students do not have the *opportunity* to learn significant mathematics'" (NCTM, 2000, p. 5 [emphasis added]). Thus, successful implementation of the NCTM Standards, or any reform curriculum, must include *opportunities* for students to participate in mathematical inquiry in the ways outlined in the *Standards*. In other words, if teachers do not give students the intellectual autonomy—to elaborate on their answers, justify or challenge others' statements or assumptions, make conjectures and mistakes, create alternative problem-solving procedures,

apply mathematical concepts to everyday events, or simply engage in open discussion about mathematical concepts—mathematics classes cannot begin to achieve the goals highlighted in the *Standards.*

Like all initial steps, a teacher's creation of opportunities for students to participate likely first will be given through discourse, either through direct instruction or through ways in which the teacher engages the students in a discussion of activity.

Method

Participants and Materials

This study focuses on four teachers, who, in their first year of being observed, relied on traditional curricula and, in their second year of being observed, used the reform-oriented curriculum *Math Trailblazers* (Wagreich, Kelso, Bessinger, & Goldberg, 1997). *Math Trailblazers,* funded by the National Science Foundation, is a standards-based K-5 curriculum.

The four teachers *(pseudonyms used),* Mrs. Cross, Foster, Jewel, and Silver, who participated in this study, are female. Two of the teachers were veteran teachers, having taught for 16 (Silver) and 21 (Foster) years. The other two were newer at teaching, each having taught for 3 years. During our investigation, they taught fourth grade in urban elementary schools. Two of the teachers, Foster and Jewel, taught at the same school. Teachers Cross, Foster, and Silver are Caucasian, and Teacher Jewel is African American. The racial compositions of the three other schools were notably different. The racial composition of the school in which Foster and Jewel taught was 77% White, 17% Black, 2% Hispanic, and 4% Asian. Teacher Cross' school had a racial composition of 90% White, 2% Black, and 8% Asian. Silver's school environment was 44% White, 15% Black, 29% Hispanic, 11% Asian, and 2% Native American.

During the summer before beginning teaching with the new curriculum, the teachers took part in 20 hours of workshops and activities to introduce them to the materials. During the academic year, they spent another 20-30 hours in further workshops and activities. This preparation was typical for teachers who began using *Math Trailblazers* during the time frame when we collected the data reported here.

Math Trailblazers is an elementary mathematics curriculum that reflects the goals and ideas of the NCTM Standards. Like *Everyday Mathematics, Math Trailblazers* focuses on real-world contexts that are easily grasped by children. It is conceptually oriented, although it does not ignore the procedural component of mathematics. *Math Trailblazers* focuses on four areas that are highlighted by the *Standards:* (a) problem solving, (b) communication, (c) reasoning, and (d) making mathematical connections between

concepts and to everyday activities. All four of these foci can be and are seen through discourse.

Data Source

We videotaped four or five consecutive days of each class's initial lessons about equivalent fractions. We did this for each of two consecutive years, leaving us with a total of 38 lessons for analysis. Originally, we observed more than four teachers, but a lack of videotaped records for some teachers forced us to exclude them from this study. We did this based on the decision that we should sample classroom lessons deeply, four or five per teacher, per year) rather than simply sampling more teachers. Additionally, we wanted to capture a teacher's approach to a new lesson. However, we designed the original coding scheme based on all of the lessons we had observed. Thus, the codes developed capture typical features we found in all classes.

Coding Procedures

In general, we tried to capture the idea that teachers have to provide ample opportunities for students to have autonomy in space, time, and thought when answering questions in the midst of other students. Because of this concern, only behaviors that required mathematical thought or participation from the students were considered in this coding scheme. For example, student questions that pertained to classroom management or the dissemination of materials were ignored by our coding scheme.

In addition, we recognized that autonomy is not simply independence, which is a student's ability either to work, study the material, or solve problems on his or her own. Because of this concern for autonomy and responsibility within a community of learners, one-on-one interaction (between the teacher and a student) cannot capture responsibility and autonomy in the ways we define them. Thus, we also omitted discussions concerning student behavior and one-on-one interactions with the teacher from our coding and only coded those that were observed during whole-class discussions. We believe that student-to-teacher interaction is informative. However, whole-class discussion provides every student access to the norms that the teacher attempts to establish. Thus, students, even when not called, come to understand in what ways a teacher's expectations, like posing a conceptually-based question or challenging a student's answer, create the intellectual space for the student to disclose either their personal problem solving procedure or their confusion. This restriction allowed us to conduct analyses and make claims about the practices to which all students had access.

Additionally, we did not note student interaction. Such exchanges were rare in these classrooms. When they did take place, many students' responses or questions were inaudible, and then, only loosely restated by the teacher. Given the scarcity and brevity of these exchanges, we did not include

them for analysis.

Most of our coding came directly from verbatim transcripts of each classroom. At the times when the two coders disagreed or where the transcript was unclear, the coders reviewed the original videotape to clarify what was said and to support the way in which they coded each lesson.

We categorized behaviors mutually exclusively as autonomy granting, autonomy diminishing, or neutral (hereinafter math-talk) with respect to autonomy granting. We coded teachers' initiations and students' responses, and, in some cases, simply, a student's attempt to respond. Originally, we identified many subtypes within each of these three major categories (see Table 1 for the subtypes we had originally identified).

Table 1. Original Coding Categories for Autonomy-Related Behaviors

Type of Autonomy	Type and Source of Behavior	Specific Coding Category	Definition
Supporting	Teacher Questions and Requests	Procedure	Requests for students to provide problem-solving procedures
		Context	Requests to generate contexts or creating examples for math concepts or problems
		Explanation	Requests for explaining math concepts or problems
		Scaffold	Provides moderate assistance during student contribution
	Student Contributions	Response	Provide a response to request for procedure, context, or explanation
		Question	Provides math-related question or suggestion
		Scaffold	Student provides scaffolding to another student
	Teacher Responses	Accept	Teacher accepts student's question or suggestion
		Compromise	Teacher negotiates a compromise with a student suggestion
		Support	Teacher helps or gets another student to help original student to solve problem on his or her own
		Encourage	Teacher provides conditions for student to solve problem on his or her own
		Switch	Student takes the role of the teacher
		Class Evaluation	Teacher asks other students to evaluate student response
		Eval Exceptional	Teacher evaluates student response as exceptional response
Diminishing	Student Contribution	Refuse	Student refuses to contribute
	Teacher Responses	Takeover	Teacher takes over during student contribution
		Passing	Teacher removes autonomy from one student, but gives it to another
		Dismiss	Teacher dismisses student contribution
Neutral	Teacher Questions and Requests	Computation	Teacher requests computational response/Student provides computational response
	Student Contributions	Request Information	Student requests more information about teacher's request

Autonomy-granting behaviors. We identified autonomy-granting behaviors as any classroom behavior, whether initiated by the teacher or student, which gave a student the opportunity to verbally or visibly elaborate on how he or she solved a problem or used problem-solving processes. In essence, these behaviors signaled to the teacher and to classmates that the participating student could either contribute as a full partner in a mathematical discussion or even initiate the discussion of new mathematical ideas and connections, thus acting as a full partner in the mathematical discourse in the classroom.

For example, autonomy-granting behaviors included asking students to create a problem describing a fraction, to explain a problem-solving proce-

dure, justify their solutions, or describe their thought processes in arriving at an answer. Additionally, autonomy-granting activities included any general positive reinforcement for such autonomous opportunities. For example, a teacher could praise a student for elaborating on an answer or attempting to scaffold another student.

Autonomy-diminishing behaviors. We defined autonomy-diminishing behaviors as any classroom behavior that took autonomy from the student once the student had been granted such. For example, a teacher may have taken autonomy from a student by appearing impatient and answering the question herself. She may have failed to allow or discouraged an autonomous event by not answering a student's question, ignoring a student's suggesting, or prohibiting a student from accepting an opportunity for autonomy. Theoretically, autonomy could be diminished by the actions of a teacher or by another student. However, no incident was observed where one student took autonomy from another student and the teacher allowed such an act. Thus, autonomy-diminishing behaviors in this sample were solely the result of the teachers' behaviors.

Math talk. We defined math talk as any classroom behaviors that were math related, but neither granted nor diminished autonomy; in other words, they were neutral with respect to granting autonomy. These behaviors generally solicited answers that required simple rote responses. Although, by design, the lessons were devoted to learning fractions, a large part of class time was devoted to reviewing mathematical operations such as addition, subtraction, and multiplication of whole numbers. Although mathematical, these contributions did not allow the students to move past responses of material previously covered and understood by many and thus we did not consider these behaviors acts of autonomy.

For example, in the context of learning equivalent fractions, the question "What is 3 times 2?" neither grants nor diminishes students' autonomy. Although such a question may be important, mathematically, it does not allow the student any intellectual space to deal with the concept and understanding of fractions deeply.

Additionally, there were times when students were given an opportunity for autonomy but, instead of taking autonomy, expressed that they did not understand the initial question or request given to them by the teacher. In some cases like this, the teacher's question was coded as autonomy-granting, but because the student did not accept nor reject the opportunity, we considered the student's response as neutral with respect to autonomy.

Eventually, we combined the subtypes for two reasons. First, certain behaviors logically fell into one of these three major categories and we could not logically privilege one type of behavior over another as more or less autonomy granting. Second, after running correlation analyses, we found that most of the subtypes within major categories were significantly related

to each other (45% of the correlations had values > .40), so empirically, we could not justify using these separately. In sum, we had no justification for treating or reporting subtypes separately, with one exception: we calculated eligibility on the subtype level because we computed reliability before collapsing the categories. We report reliability at this level to underscore the point that the particular types of autonomy-granting behaviors could reliably be noted.

One coder reviewed and coded 100% of the lessons. An independent coder examined 25% of whole-class instructional behaviors in the 38 lessons. Cohen's (1960) kappa for coding behaviors was 0.82. Reliability calculated using simple agreement was 0.89.

Results

First, we present a quantitative analysis of the data, highlighting the frequency of autonomy-granting behaviors in one year (the traditional-curriculum year) and in year two (the *Math Trailblazers* year). In this section, we compare teacher behaviors between year one and year two. We compare teachers as a group. After that, we report a qualitative analysis of the data, focusing on both teacher bids and student responses to opportunities for autonomy. We highlight specific ways in which the autonomy-granting behaviors changed from one year to year two. Our qualitative approach juxtaposes teachers' individual lessons from year one to year two.

Guiding Questions

We began by wanting to know how often the teachers in our sample, when using a traditional curriculum, employed discourse practices that could provoke intellectual autonomy in their students. Next, we wanted to know if the frequency of such discourse practices changed as each teacher moved from a traditional curriculum to a reform-inspired curriculum. Finally, if they did change, what did this change look like? Did the change reflect the suggestions advanced by the NCTM Standards?

Difference Across the Years

Initially, we expected that the teachers in our sample, when given a reform curriculum, would increase, through verbal exchanges, the amount of intellectual space they granted to their students during whole-classroom instruction. We expected this because reform curricula in general, and the specific curriculum these teachers adopted, endorse the position that students should have intellectual autonomy and responsibility for contributing mathematical ideas. Thus, we asked: When moving from year one to year two, did we see these teachers create more opportunities for autonomy

among their students? Or, did the amount of opportunities for autonomy stay the same?

Autonomy-granting behaviors. The average number of autonomy-granting behaviors for the four teachers in our sample increased from 105.25 behaviors in year one to 159.25 behaviors in year two (see Table 2 for descriptive data and Table 3 for descriptive analyses). Using a repeated measures ANOVA, we found that this increase across years was significant, $F(1, 36) = 4.39$, $p = .04$ (see Table 4), with impressive effect sizes (eta square = .825). These teachers, as a whole, used more autonomy-granting behaviors when teaching fractions from *Math Trailblazers* then when they used a traditional curriculum.

Table 2. Average Number of Autonomy-Granting Behaviors — Teacher by Year

	Year 1		Year 2	
Teacher	*M*	*SD*	*M*	*SD*
Cross	30.60	13.93	34.60	18.47
Foster	6.60	5.77	22.60	6.76
Jewel	26.60	23.80	43.00	13.28
Silver	25.50	19.95	31.70	14.31

Table 3. Descriptive Statistics for All Autonomy-Granting Behaviors

	Year 1		Year 2	
Year	*n*	*Sum*	*M*	*SD*
Year 1	19	421	22.16	18.38
Year 2	19	637	33.53	14.84

Autonomy-diminishing behaviors. Autonomy-diminishing behaviors rarely occurred. In fact in the first year, among all four teachers, we only observed nine autonomy-diminishing behaviors. In year two, we only observed 14 autonomy-diminishing behaviors. Additionally, autonomy-diminishing behaviors did not occur in all classes. Because the number of autonomy-diminishing behaviors was so low both in year one and two, we did not perform any statistical analyses on these data.

Table 4. Analysis of Variance for Autonomy-Granting Behaviors across Year 1 and Year 2

Analysis of Variance for Autonomy-Granting Behaviors across Year 1 and Year 2

Source of Variation	*SS*	*df*	*MS*	*F*	*p*
Between Groups	1227.79	1	1227.73	4.39*	.04
Within Groups	10051.26	36	279.20		

Math talk. The average number of math-talk behaviors for all teachers increased from 231 behaviors in year one to 299 behaviors in year two, $F(1, 8) = 6.47$, $p = 0.03$. Although, collectively, there was an overall increase in math talk from year one to year two for two teachers (Jewel and Foster) only one teacher, Jewel, had a significant increase in math talk. For Cross, there was no significant increase or decrease in math talk. Although not significant, for Silver there was a slight decrease in math talk from year one to year two: 68.75 instances, on average, in year one, compared to 53.5 instances, on average, in year two.

Differences Among Teachers

Although all of the teachers used a traditional curriculum in the first year, only the two teachers who taught in the same school, Foster and Jewel, used the same textbook during that first year. Given this, we wondered whether teachers were more dissimilar to each other with respect to providing more intellectual opportunities in year one, when working with different textbooks, than they were in year two, when each was working with the same curricular materials. Essentially we asked if the introduction of a consistent reform-based program in year two compelled these teachers to teach in a more similar fashion. Thus, we compared these teachers to each other in year one, and again to each other in year two.

To accomplish this analysis, we compared the variances in both autonomy granting and math talk between year one and year two, using an F-test for two sample variances. We found no statistical differences in variance for autonomy granting, $F(3,3) = 1.21$, $p = 0.43$, or neutral behaviors, $F(3,3) = 1.68$, $p = 0.47$. Thus, the amount of similarity across teachers with respect to autonomy-granting behaviors did not change when adopting this reform mathematics curriculum. This suggests that teachers retain their individual styles of teaching, making adjustments accordingly, when using a new curriculum.

A More Qualitative Understanding

Our quantitative analysis showed differences in autonomy-granting behavior used by these four teachers from year one to year two. In addition, after looking extensively at the videotapes, it became apparent that the *way* in which autonomy-granting behaviors were employed in year two was different than how they were employed in year one. Having viewed these videotapes many times, we were surprised by the difference in the quality of questions and explanations in year two, as opposed to year one, especially for Foster and Silver. These teachers, at times, asked more conceptually-based questions or they challenged the students to disclose more of their thoughts and strategies.

We did note similar differences in the other two teachers, but they were

not as striking. Although the students' responses in year two are similar to those in year one, the quality of teachers' questions are different. Additionally, we did witness slightly longer student responses and more discourse turns spent on any one student's answer, when responding to a teacher's "autonomy-granting" question. We will attempt to illustrate differences between how Mrs. Foster and Mrs. Silver differed from year one to year two.

The selection of these two teachers was logical because they both showed notable increases in autonomy-granting behaviors when using *Math Trailblazers.* We found that Foster used an average of 16 more autonomy-granting behaviors per lesson in year two than in year one and Silver used an average of 6 more. Although not as dramatic as Foster, the autonomy-granting behaviors in Silver's class took on a deeper quality in year two than those we observed in year one. Thus, in this section, we report how autonomy-granting behaviors were manifested by these teachers when using a reform curriculum.

To determine how autonomy-granting behaviors were used across years one and year two by Foster and Silver, the videotaped segments of all autonomy-granting behaviors were reexamined. In particular, we asked: How did year one differ from year two, with respect to the types of questions and explanations exchanged between teacher and students? To answer this question, we elaborate on distinctive features of the questions and explanations in year one and then move to contrast noticed differences in questions and explanations in year two.

Foster and Silver's Year One

In year one, both Foster and Silver asked a lot of leading questions, typically directing and controlling verbal exchanges. Additionally, even when they asked a question that could have granted autonomy, they would often answer it themselves, thereby mitigating the opportunity to grant autonomy to their students. The bolded statements, in the following excerpt, taken from Foster's first day of class, in year one, could have been opportunities to encourage students to disclose their thoughts, confusion, or problem-solving procedures.

Teacher: So, let's take this one, for instance. If we started with the 6/12 and remember where it is that we wanted to go to? How do we get there? Lana says we should divide. What should we divide by Lana?

Student: Uhm, you divide...6 divided by 6 and you get 1.

Teacher: All right, so if we divided this divided by 6 and this divided by 6, we would get...

Student: 1 and a 2.

Teacher: A 1 and a 2. **So 6/12 is exactly the same as 1/2.** Is that what we've already said in this line up here? We've already said

that all of these fractions are equivalent. **Did you know that that little symbol right there, that means "equivalent"? It means equivalent to 2/4 or 3/6 or 4/8 and so forth.** OK. So, now today, ooh, today, we have a fancy little booklet that looks like this and this is one of the things for which you will either need....ooh, we suppose we should really do the 1/3 and the 1/4, shouldn't we before we hand these out? OK. Let's do that. All right, let's get all our statistics in here. OK. Here we go. 1/3. Somebody start me off with 1/3. Let's see, uhm, Anna.

Student: OK. 1/3 = 2/6.

Teacher: **Good 1/3 is equal to 2/6. Once again, if we took this number and multiplied the numerator times 2 and the denominator times 2.** Aha. We'd get 2/6. Give me something else that's equivalent to...Alexander.

Student: Uhm. Like 3/9.

Teacher: **How about 3/9? Multiply times 3, times 3.** We've got it. Give me another one. Ann.

Student: Uhm. 4/12.

In reality, these teachers' statements replaced the possible thinking on the part of the students. What could have been a discussion between teacher and student (or student and student) was, in reality, more of a lecture. Essentially, the teacher provides the critical mathematical concepts in the lesson, although the students could have done the same. For example, the second sentence in fifth turn could have been supplied by a student if asked either how or why she/he arrived at the numerical answer. Consequently, students were not given the opportunity to struggle with the material, elaborate on their answers, evaluate other student responses, or engage in the process of mathematical thinking. The routine of the classrooms was the acceptance and retention of information presented by the teacher, and restricted students' opportunities to engage in the process of mathematical thinking. The following excerpts represent the type of discourse we witnessed in the first year for each teacher.

In Foster's class, we observed the teacher first ask and then answer many of her own questions. The only real explanations were given by the teacher. Additionally, her questions led the students to the correct answer; they did not allow the students to discover the correct answer through individual reasoning, or even guessing. When the student answered a question, only one- or two-word answers were offered, rather than elaborate responses or explanations. Thus, the students in class did not have the opportunity to engage in the material in any mathematically engaging way.

The following two excerpts represent the type of dialogue in year one of these teachers' classrooms. The first is from Foster's second day of class, and the second is from Silver's first day of class.

Teacher: Danny, what's....1/2 is equal to what?
Student: Uhm....2/4.
Teacher: 2/4. Yes.
Student: And, uhm..
Teacher: And what else?
Student: 3/6.
Teacher: 3/6. And what else David?
Student: 4/8
Teacher: 4/8. And what else, Alexander?
Student: Uhm, 4/8 and uh...5/10.
Teacher: And so forth. You're going to go along until you've given me about 10 things. Rachel M., did you get it? You're going to write down about 10 equivalent fractions for 1/2. Now, do you suppose there's any tricky way to get this in case you forgot?
Student: Yeah.
Teacher: **Well, you could multiply. You guys are good on your multiplication facts, so you could take that top number and you multiply that times 2 and the bottom one times 2 to get that first one.**
Student: 4.
Teacher: The next one after that, **you could multiply the top times 3 and the bottom times 3 to get the one after that. Because if you do the same thing to the numerator as you do to the denominator, it's alright. You haven't changed the value. You got that?** OK. So, then you could multiply times 4 x 4 x 5 x 5 x 6 x 6 and keep going. Do you think you could also do that for 1/3 and 1/4?

From Silver's first day of class:

Teacher: If we times 3. Excuse me. **If we do it in the denominator, we have to do it in the numerator.** You have to have the same number to equal. So, 4 times 3 is 12. What do we have to do at the to end?
Student: (inaudible)...we mean...2 times 3.
Teacher: We have to use what?
Student: 3
Teacher: 3. Don't we? So, 2 times 3 is...

Students: Six.
Teacher: 6. **We don't write this but we think it! Sometimes, if you wanna write it to think it.** Okay.
Student: 5...times...(inaudible)
Teacher: What do you think about this one?
Students: 5 times 3
Teacher: Wait. Wait. Wait. Wait. Wait. Let's see if Fabio. Fabio. We know 2...5 times something equals ten, right?
Student: Right!
Teacher: **What would be the common factor that we could use for...to make something here? Five times what equals ten?**
Student: (inaudible)
Teacher: What do you think...
Student: 5 times 2.

Similarly, in Silver's class in year one, students were given relatively few opportunities for autonomy. The excerpts we have highlighted are typical of the five lessons we observed during year one. In Silver's class we saw that the teacher and the students were satisfied with one- or two- word answers from the students that, often, had little to do with fractions, ostensibly the intended mathematical concept to be grasped. Instead, we saw a concentration of questions concerning the multiplication of whole numbers; although mathematical, this material did not require deep thought on the part of the students. In fact, at this grade level and throughout this excerpt, it is obvious that multiplication is a concept that had already been grasped. Thus, students were not expanding on, struggling with, or discovering new aspects of fractional concepts. Because of the type of discourse, students were restricted from engaging with the material on a meaningful level.

In year one, for both of these teachers, students were given little opportunity to elaborate on their answers. In fact, it seems as if students did not expect to elaborate on their answers, apparently recognizing that the teacher and class were satisfied with short answers. The teacher quickly jumped in after a student had provided an answer, thus eliminating the possibility that a student could provide a justification, elaboration, or explanation. Clearly, students were not expected to explain their problem-solving procedures nor where they expected to comment on someone else's procedures. The atmosphere and expectation of the classroom was one in which students were expected to answer simple math questions, which seemed barely related to the learning of fractions.

Foster and Silver's Year Two

Excerpts from Foster's and Silver's year two observations, when they

were using the reform curriculum, *Math Trailblazers,* highlight the ways in which they changed. We noticed a substantial difference in their discourse practices, which was likely a direct result of the introduction of the new curriculum. The following is from Foster's first day of class in year two.

Teacher: Do you know, Alex? How can we do it?
Student: We know the answer.
Teacher: Don't give me the answer. Tell me how to do it.
(Note that Foster asked for this student's thought process and not for an answer. We almost never saw this type of questioning in year one. The excerpt continues:)
Student: Uh. You sort of add 1/2 with 3/4.
Teacher: How? How can we?
(Here, she continued to challenge the student to elaborate. She did not offer leading questions; instead she waited for the student to give his own reasoning.?
Students: (inaudible)
Student: Oh. Uhm. We could...the 1/2 is the 1/2 and then 3/4 is like it's over one half, sort of and then uhm, when you add them together, they're over one whole.
Teacher: Yes! They are over one whole. Yeah. You're absolutely right, Alex. We wanna get a little strategy here that we could all use, when it's time to do this. Help, Jackie. Help me!
(Foster moved to draw other students into the exchange. Students were expected to think and explain their problem-solving procedures (or that of others) instead of giving a simple one or two word answer.)

From Foster's second day of class in year:

Student: Frank is baking a cake. The recipe calls for 1/4 cup of oil and 3/4 cup of water. How much liquid will Frank add to the cake? I put 1/4 plus 3/4 equals 4/4ths.
Teacher: Thank you! Did anybody else put anything different besides 1/4 + 3/4 = 4/4ths? Which is correct but you can write it another way. Kathleen, how did you write it?
Student: Uhm 1/4 plus 3/4ths = 1 whole.
Teacher: Equals one whole. How many put either what Kathleen said or what Julia said? Good! They are both correct. Thank you! Excellent: Put your hand down. Mina! It happens to be you next.
Student: Okay. Jessie used 5/12ths...(inaudible)...what fraction of the...(inaudible)...for another project? I put uhm 12...a whole

minus 5/12 = 7/12ths!

Teacher: Okay! What you started to say at first, Mina! Tell me again! How else could you write one whole?

Student: 12/12ths.

Teacher: And in this particular case because they want you to take away 5/12ths, it's a pretty good idea to start with the one equaling 12/12ths. Right? Okay! How many people decided to do just what Mina did and change that one whole into 12/12ths? So! You could write down 12/12ths. Good! Put your hands down. 12/12 — 5/12ths was what again?

Student: 7/12ths.

Student: 7/12ths.

From Silver's first day of class:

Teacher: The... What?... What? How can we? What can we? What can we figure out from this?

Student: (inaudible)...the denominator.

Teacher: Roxanne, you've got a good point! Sean. What can we figure out from this? If the denominator...

Student: If the denominator is smaller, then the number is extra bigger.

Teacher: Wait. Wait. Wait. I'm confused. If the denominator is smaller...?

Student: Like if it was on the bottom...

Teacher: The denominator.

Student: If it was a 2 and on the top it was...

Student: the de...numerator

Student: A one then it would be half and then that would be bigger than uhm like than the 1/12th...or...anything

From Silver's first day of class:

Teacher: What do you think? Zed...Uh! Roxanne?

Student: One!

Teacher: One? How did you figure out one?

Student: 6/12ths equals, is equivalent to one half.

Teacher: How did you figure that out?

Student: By...put 6...(inaudible)...when you...I got the chart.

Teacher: The chart? You figured out by the chart? You noticed it? But what happens if we have numbers on the chart that we can't figure out

Students: I know! I know!...Ms. Silver!

Teacher: What could we do? Could we have...Candace!

Student: 6 is half of 12, so...

Teacher: ...6 is half of 12 so, you know it's one half. Or! What's another way? Samantha?
Student: inaudible
Teacher: Shh! Here's Samantha! Listen!
Student: You could think of the problem.
Teacher: Think of the problem.
Student: 12 minus 6. And you get...
Teacher: Or...
Student: And it's half!
Teacher: Okay! You could figure out half! Anyone else? Okay! Let's try!

In this second year, Foster challenged the students more. Not only was the manner in which Foster approached problems different, the student responses were also different. They were longer than a few words or mere phrases. We found similar results in Silver's class. In year two, Silver asked questions that made her students explain and justify their answers. No longer was she or the class satisfied with one- or two-word answers pulled from a student's memorized arithmetic facts. In the second year of observation, all of the teachers, and especially the two whose lessons we have highlighted, appeared adept at engaging students to participate in mathematical discourse. The teachers made it mandatory that students reveal their thinking and thereby act autonomously in these fourth-grade mathematics lessons. We also witnessed more scaffolding in year two. Both teachers probed students' thinking by asking them to justify and elaborate on their answers.

Discussion

Because *Math Trailblazers* is a NCTM Standards-based curriculum, we assumed that these teachers, when using this curriculum, would employ many of the principles of the NCTM Standards concerning classroom discourse. Although we could have anticipated various changes, we focused on our investigations on the opportunities that teachers provided for intellectual autonomy both before and after adopting *Math Trailblazers.* Collectively, these four teachers significantly increased the amount of autonomy-granting behaviors they employed in their classrooms when using a reform curriculum.

For the two teachers whom we highlighted, we witnessed a dramatic difference between both the types of questions asked and the types of responses solicited by those questions in year one compared to those in year two. We found that the questions in year two were designed to elicit longer and more thoughtful responses from the students. In fact, in year two, the stu-

dents' answers and explanations were generally longer and involved more elaboration on the part of the student. We also noted similar changes in the other two teachers' classrooms. Given the introduction of a new curriculum, we attribute this change, in part, to the adoption of this curriculum.

We believe that when teachers adopt a reform-oriented curriculum, which focuses on conceptual understanding, they will alter their approach to teaching mathematics. Namely, their questioning will focus more on students' understandings and misconceptions. We witnessed the beginnings of such a transformation in year two of this study. Although many of the teachers' questions were still procedural, they were qualitatively different than the questions in year one. Students felt compelled to give longer and more thoughtful answers.

Because this investigation did not focus on the problems or structure of the textbook, we cannot suggest that this change was a direct result of the instructions in the manual. However, we are confident that these teachers, in attempting to align themselves with the curriculum, changed the way they questioned their students and allowed these students to handle the material.

In year two, we witnessed teachers asking questions related to students' informal knowledge, personal experience, or mathematical-emergent point of view. In other words, if *Math Trailblazers* presented problems that were focused on the students' possible experiences, then teachers, following suit, would have to elicit more personal responses from students, giving rise to discussions that focused on the concepts in and connections between mathematical topics. Although we do not have this exact evidence, we realize that it is possible that this reform curriculum provided both an impetus and a mechanism for implementing pedagogical change. Given the changes that we witnessed, we suspect that reform curricula can be a viable first step in changing the dynamics of a mathematics classroom, where teachers are encouraged to initiate and promote intellectual autonomy in their students.

We found that this reform-oriented curriculum moved teachers in a direction that is more aligned with principles of the *Standards* than what we saw with the curricula they used in year one. But a curriculum, alone, may not cause a significant change in the way teachers approach elementary mathematics. Teachers who have become comfortable with traditional methods and curricula probably resort to traditional approaches and resources when they are faced with students' confusion, complex student questions and suggestions, or unfamiliar ideas presented by a reform curriculum (Clift & Brady, 2005). Furthermore, teachers may not have a firm understanding of each of the unit's goals and ultimate congruence with other parts of the lesson (Remillard, 2005). Thus, we reiterate that a curriculum can provide an excellent starting point for improving mathematics teaching and learning, but, especially with entrenched beliefs, this is only a start.

Although a reform-based curriculum could engender some changes in classroom dynamics, we would likely witness a more thorough and effective change if teachers' beliefs about discourse in mathematics classrooms were examined and realigned. When adopting a new curriculum, ideally schools would provide teachers with systematic training not only on the new curriculum itself, but also the principles and goals of the reform curriculum. For curricula like *Math Trailblazers* and *Everyday Mathematics,* it may be necessary to inform teachers about the suggestions of the NCTM Standards*,* in large part because the structures of these curricula are informed by the suggestions of the *Standards* and it is hard to imagine producing a Standards-based classroom without some groundwork in the Standards.

Additionally, we suggest that teachers watch videotaped lessons that highlight some of these changes in practices. Being able to witness the difficulties and incremental changes a teacher *can* make, would allow preservice and veteran teachers to understand what change looks like in real and practical settings.

Hopes for Future Investigations

Future studies should focus on what aspects of classroom discourse positively effect students' learning outcomes. There are times when teachers need to be didactic, but this form of instruction should not dominate the discursive dynamics of a mathematics class as students struggle to become autonomous students of mathematics. Thus, future studies should examine the length and complexities of discussions, the amount of time given to particular topics, and who does the talking (Sims, 2008) with the aim of understanding just how different modes of discourse support different kinds of learning, especially under circumstances where teachers are more or less adept with the curriculum and mathematical content.

Final Thoughts

The *Standards* call for teachers and students to engage in a mathematical dynamic far different from what has been traditionally accepted. To elevate U.S. students to true and mathematical success, embracing reform curricula and the suggestions of the *Standards* appear to be a step in the right direction. They offer a conceptual platform for teachers to make mathematics a personally reachable and valuable subject. In this study, the teachers we observed showed a change in the way they approached mathematics and how they presented the materials to their students when they used a reform curriculum which we did not see when teachers relied on a traditional curriculum.

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Teaching Mathematical Connections to Financial Literacy in Grades K-8: Clarifying the Issues

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Abstract

Most teacher education programs do not incorporate financial education preparations into courses required for early childhood, elementary education, and middle level candidates. The authors of this manuscript explore the reasons for this omission, particularly the mathematics education component, and clarify the issues surrounding this decision. They argue that financial education represents a valid curriculum concern and that inadequate personal finance literacy and mathematics standards exist. In addition, they discuss that elementary and middle schoolteachers generally lack understandings of the content, that teacher preparations inadequately educate teachers to teach about this content, and that a pedagogical shift is needed to affect meaningful change. They call for revisions of traditional mathematics pedagogies to address this challenge so that K-8 students may begin to exercise the responsibility for prudent financial decision-making that they will need in future years.

This paper presents a justification for a stronger presence of financial education curricula in grades K-8, particularly in the mathematics methods components of elementary education, and middle level teacher education programs. Reform of teacher preparation programs is needed to improve the K-8 candidates' understandings of personal finance and its mathematical underpinnings. Revising standards, modifying teacher preparation, and reconsidering pedagogy are needed for this change to occur and are addressed in the following manner through his paper. First, concerns about

financial illiteracy as an educational matter and legislative inattention and superficial primary and elementary grade coverage are addressed. A discussion about the mathematical connections to financial education ensues. The third section relates matters of teacher preparation and their associations to educating teachers about mathematical and financial literacy matters. It also provides recommendations for improving these efforts. The final two sections call for revisions of traditional mathematics pedagogies to address this challenge so that K-8 students may begin to exercise the responsibility for prudent financial decision-making that they will need in future years.

Financial Literacy as an Elementary Education Concern

Financial literacy's place in elementary and middle school depends on the philosophies of those who interpret the problem. Legislators recognize the need for financial education; however, they have historically focused on learning processes in middle and upper grades (National Council for Economic Education, 2005; 2007; The Council on Economic Education, 2009). Consequently, elementary school children lack basic understandings of basic economic and financial knowledge (Lucey, 2002; McKenzie, 1970). If early childhood and early elementary education lay the cognitive and behavioral foundations for future learning, then a formal financial education curriculum represents a critical goal for these children. In essence, children are caught between legislative demands and developmentally appropriate pedagogies.

Arguably, formal instruction of all financial education tenets may not be appropriate for elementary and middle age children; however, some developmentally appropriate tenets of financial literacy are appropriate for all levels. Schug and Birkley (1985) conclude from their interviews of 70 randomly selected urban elementary school children that economic knowledge develops through experience. They indicate that formal economic instruction should begin at the "upper primary or the intermediate grades" (p. 41). Yet, there are no national mathematics standards that exist to connect directly with the teaching of financial literacy, in spite of the mathematics reform efforts instituted during the last two decades (National Council of Teachers of Mathematics (NCTM), 2000).

Literature (e.g., Johnson & Sherraden, 2007; Lucey, 2002) tends to support the belief that children should learn the basic financial education tenets of financial responsibility and decision making, income and careers, planning and money management, credit and debt, risk management and insurance, saving and investing by Grade 4. Examples of these tenets may be found in the Jump$tart Coalition's National Standards in Personal Finance (http://www.jumpstart.org/assets/files/standard_book-ALL.pdf); however, these standards do not provide any specific mathematics expectations. As children develop, problematic mathematics pedagogies compound these

challenges. Brenner (1998) points out mathematics curricula fail to teach financial principles consistently with patterns of child development. The labeling of elementary and middle level mathematics curricula as "incoherent, cursory, and repetitive" (Silver, Messa, Morris, Star, & Benken, 2009, p. 503) provides a grim, yet realistic, descriptor that explains the inadequacies of mathematics coverage in schools.

The mathematics connections to financial education in elementary and middle grades also suffer from developmentally inappropriate curricula. Children naturally develop knowledge of coins and currency through relational interpretations. Curricula focus on conceptualizing coin values. Furthermore, younger children do not conceptually understand decimals numerals, even when representing monetary amounts (Van de Walle, 2007). Under pressure to ensure that children learn content within a prescribed amount of time, elementary teachers may spontaneously develop class examples that are inappropriate for their students' development. Anecdotal evidence indicates that primary grade teachers (K-2) frequently use symbolic representations such as $1.45 when giving examples of the use of addition and subtraction of dollar and cents amounts. From a mathematics perspective, this amount is an erroneous representation of a rational (fractional) number, often presented prior to a formal introduction to rational number operations. In primary grades money should be represented in terms of dollars using $ symbol, and/or cents using the ¢ symbol exclusively. For example, a second grade teacher could ask the total cost of items whose individual costs are 25¢ and 45¢, or $12 and $5. Teachers could use the combination only when they introduce fraction understanding (estimation in Grade 3) and computation through decimal numerals in grades 4-6.

Curricula should also delay the topic of compounding interest until students understand exponents in grades 6-8 (Van de Walle, 2007). The middle school mathematics teacher should understand how to derive the financial formulas in order to help his or her students learn conceptually about the significance of exponents (Dworsky, 2009). Although it is not advisable to demonstrate the entire process resulting in the formula in the middle school classroom, the students can manually calculate several iterations, and then move to a computer or preprogrammed spreadsheet to explore how the changes of the various components can affect the outcomes. This example shows the link between the knowledge of mathematics and the pedagogical tactic of exploration coupled with discovery.

Inconsistent state efforts to implement national personal finance curricula exist. According to The Council for Economic Education (2009) 44 states disclose that they have developed guidelines for financial education in grades K-12, with 34 requiring implementation of these standards, and nine requiring student-testing. Of the 44 states indicating development of standards, 29 (AL, AZ, GA, HI, ID, IL, IN, IA, KS, KY, LA, MN, MD, MD,

MS, MI, MT, NB, NV, NH, NJ, ND, OK, OR, PA, SD, VT, VA, WV, WI) indicate that they include elementary grades in their guidelines.

Historical Connections to Mathematics

In the past, financial education was connected to mathematics as an effort to teach life skills through rudimentary mathematical process. During the 1960s through the 1980s, students were often tracked into mathematics courses by their mathematical abilities. Teachers taught personal finance education, know as consumer education, to the mathematically lower tracked students (Useem, 1991). These courses emphasized simple addition, subtraction, and multiplication processes associated with routine consumer receipts and expenditures.

When the mathematics reform movement started in the 1980s, those tracked school programs faded away as educators stressed equality of learning for *all* students (Morton, 2005). Additionally, national standards were developed by mathematics educators promoted from an examination of content equity (Morton, 2005). Teachers were advised to present mathematics connections to real life situations through motivational emphases or project-based learning opportunities. In the quest to teach *all* children mathematics equitably, mathematics teachers who employ the pedagogies initiated by the reform efforts have decreased the amount of consumer mathematics taught (NCTM, 2000).

When the majority of these teacher candidates learned mathematics (during the 1980s-1990s), many programs lacked the mathematical instruction to give these candidates adequate financial underpinnings. Although educators are concerned about each learner developing a conceptual understanding of mathematics, this process is dependent on both the curriculum and knowledge of the teacher. Unfortunately, the inclusion of the consumer education topics (if present) is scattered through commercial textbooks. Maxwell's (2008) description of the history of mathematics education and its failure to completely embrace tenets of personal finance illustrates the difficulties of promoting equitable mathematics learning efforts through practical frameworks.

It appears that coverage of personal finance within mathematics curricula has historically been limited to basis consumer decisions that employ rudimentary mathematical processes. This situation may relate to findings that most children learn about money at home (e.g., American Savings Education Council (ASEC), 1999; Mandell, 1998; 2002; 2004). Public schools may have provided limited information about advance mathematical connections to personal finance because this information was not deemed important or necessary for those who learned the principles of high finance in their homes. Consumer finance was included in curricula as a basic life skill for those in need; however, sophisticated financial tenets related to invest-

ments and credits could be taught at home or in elite institutions.

Teacher Knowledge and Preparation

Effective teaching involves both content knowledge and successful instruction; yet it also requires knowledge of the students and the contexts from which they derive (Bransford, Darling-Hammond, & LePage, 2005). Elementary school teachers tend to lack the understandings to effectively teach personal finance. Research (McKenzie, 1971; McKinney, McKinney, Larkins, Gilmore, & Ford, 1990; Way & Holden, 2009) indicates that elementary and middle schoolteachers and preservice teachers lack knowledge of economic (including financial) content and are uncomfortable teaching it. Elementary and middle schoolteachers may be tempted to address economic concepts lightly, or skip them entirely, because of their own financial ineptness (Schug & Hagedorn, 2005).

The challenge of elementary teachers' weak mathematical understandings compounds this situation. Financial literacy requires awareness of mathematical processes to illustrate the long- and short-term mathematical consequences of financial decision-making for children. But Goulding, Rowland, and Barber (2002) report that weak mathematical understandings occur among preservice elementary teachers. Elbaz (1983) writes, "The single factor which seems to have the greatest power to carry forward our understanding of the teachers' role [in the classroom] is the phenomenon of teachers' knowledge" (p. 45). Likewise, Ball (1990) posits that

> Teachers must understand the subject [mathematics] in sufficient depth to be able to represent it appropriately and in multiple ways—with story problems, pictures, situations, and concrete materials. They [teacher candidates] need to possess the flexibility in their knowledge to interpret and appraise students' ideas, helping them to extend and formalize intuitive understandings and challenging incorrect notions. (p. 458)

These quotes describe the sophistication of knowledge that elementary teachers need to have to facilitate effective dialogues about content with their students. Unfortunately, their shallow understandings become evident in their expectations for student learning. Silver et al.'s (2009) findings that lessons contained in portfolios of teachers seeking NBPTS certification contained predominately lower cognitive thinking learning, illustrate the thin mathematics content knowledge that elementary teachers possess. If a teacher does not own the knowledge sophistication to engage his or her students' inquiry into the math-money connections, he or she may impair learners' potential to be financially literate. The mutual learning benefits are borne out by research, which "supports the idea that knowledge is retained or useful only when mathematics learning is coupled with an understanding of mathematical processes, or when learners are able to build relationships between what they already know and new ideas" (Fennema & Franke, 1992,

p. 152). As pointed out by Hamburg (2009), "Financial literacy education needs to be permeated throughout all grade levels and all classes, especially mathematics" (p. 174). Financial illiteracy among children relates to the simplistic mathematics conceptions among elementary teachers.

The solution to increase understanding does not appear to lie in adding course requirements, but in the nature of the instruction and content within these courses. Murray, Raths, and Zhang (2004) provide evidence that knowledge of content does not necessarily correlate with the amount of mathematics courses taken by college students and their achievement. In general, elementary and/or middle school teachers who obtain a conceptual knowledge of specific content will more likely structure classroom learning activities to help share this knowledge with the students (Ball, 1990). For example, if a teacher understands compound interest, he or she can recognize the cumulative benefit of regularly saving money, thus recognizing the importance of teaching this concept. Since most elementary teachers lack an abundance of mathematics courses, the grade level textbook serves as the primary source of the teachers' curricular understanding. Hamburg (2009) concludes that because of the deficiencies of textbook financial literacy content coupled with teachers' lack of tasks requiring higher order thinking skills, educators (both inservice and preservice) need to experience and create activities that focus on personal finance categories and challenge students' thinking.

Thus, the problem with the elementary teachers' understanding lies not only within the "facts" of financial literacy, but within its depth and substance. Lucey and Cooter's (2008) portrayal of financial literacy as a multidimensional concept that requires scholarly contributions from various academic disciplines signifies the conundrum that elementary teachers face. For teachers of these grades to develop the content knowledge to authentically educate children about their personal finances, they must be prepared through engagement in the various facets of financial literacy and draw from them to reinforce the learning of these concepts.

The mathematics associated with personal finance presents a challenge for elementary teachers to relate to their students because of these dimensions. Lawrence Dworsky's (2009) *Understanding the Mathematics of Personal Finance: An Introduction to Financial Literacy* contains mathematics topics pertinent to educators' understanding of the relationship between mathematics and finance. As a series of numbers without experiential references, financial calculations are nonsensical to children. The contexts from which which children derive present teaching and learning challenges that require instructional attention. Maxwell's (2008) content analysis of mathematics textbook series for grades 6-8 found that most "financial" tasks do not incorporate financial standards. For example, *Math Connects Concepts, Skills, and Problem Solving: Course 1,* a sixth grade textbook, presents the

following problem:

> 19: Gary and Paz together have $756. If Gary has $489, how much does Paz have? Write and solve an additional equation to find how much money belongs to Paz. (Day et. al., 2009, p. 647)

This problem has no financial standard to provide a learning context. It constitutes a disguised addition problem that uses a financial context. This situation represents the crux of the problem connecting mathematics and financial education. Postman and Weingartner (1969) describe the subjective and interdisciplinary nature of subject content and the necessity of interactive strategies to illuminate its nature. As a solitary concept that emphasizes mechanical truisms and independent number processing, mathematics becomes an isolating experience where one's achievement relates to his or her determination of narrow outcomes. Taken in this thin sense, mathematics does not align well with other content areas. Rather, it represents an eclectic assembly of number processes that currently pollute K-8 curricula.

A broad perspective of mathematics provides a more authentic approach because it associates the technical knowledge with real world applications that practice the problem-solving skills that children need to perform as adults. Thus, it involves an emancipatory curricular philosophy that is relevant to the needs of all (Lucey & Lorsbach, in press). The cooperative learning process informs students that, while numeric solutions should be accurate, diverse viewpoints represent necessities for considering the possible factors (and their weights) to be evaluated in finding solutions to real world problems. This cooperative process deepens the learning. For example, reconsidering the aforementioned problem, Gary and Paz together have $756, but they may possess different intentions for its use. Gary may want to purchase some technologically advanced entertainment equipment and have a party while Paz would rather use the funds to fly out of the locale and discuss a social concern with a governmental representative. Students would examine the costs of different ways the funds could be allocated and provide various forms of argument (e.g., mathematical, logical, social, environmental) to justify findings.

Improvements in Preparation

An overdue strategy for improving financial literacy concerns the preparation of K-8 preservice teachers and inservice teachers in related curricular and instruction tenets. Research (e.g., Bosshardt & Watts, 1 994; Lucey, 2008; Schug, Wynn, & Posnanski, 2002) indicates that such endeavors offer potential for success. Research literature (e.g., Schug & Butt, 2006; Schug & Niederjohn, 2006; Schug, Wynn, & Posnanski, 2002) points to successes with teaching middle and high school teachers about personal finance; however, there appears to be scant evidence of research into similar programs

that prepare elementary school teachers. Lucey (2008) documents some success with a small sample of preservice teachers' research of standards and creation of lessons; however, much more work with larger and diverse samples is needed to confirm or deny these outcomes.

Not all endeavors generate positive outcomes. Lucey and Maxwell (2009) report that preservice teachers confidence in teaching the mathematical tenets of personal finance *decreased* after they received instruction about the topic. Preservice elementary teachers do not possess firm conceptual understandings of mathematics. Teacher preparation programs simply do not offer the courses or opportunities necessary to give their elementary and middle school candidates the self-assurance to teach personal finance content or to relate it to curriculum standards.

One may find the benefits in the use of active learning processes that engage students in dialogues about personal finance topics. Since Kourilsky and Murray (1981) document positive outcomes from dialogues and feedback of middle class parents and children about family budgeting processes, experiential learning environments may represent vehicles for both K-8 teachers and students to learn about personal finance. Berti, Bombi, and De Beni (1986) conclude that children need teachers to both provide content and to facilitate discoveries of "mature" economic ideas, such as profit; however, the nature and accuracy of the provided content affect the information discovered. But Morton's (2005) argument that unprepared teachers of personal finance may intentionally or unintentionally teach content that contradicts economic principles or may ignore the topic completely suggests little margin for critical interpretation of related principles. A mechanized mathematical approach to financial literacy that blindly accepts numerically-based relationships among populations yields a narrow view of content. Critically minded discussions that originate from socially-based and mathematically intensive problem solving and that encourage dialogues about statistical assumptions are needed to produce and develop students' broad conceptions of mathematics. Just as teacher candidates are constrained by their narrow views of citizenship and democratic tenets (Carr, 2008; Lucey, in press), they are also hampered by linear conceptions of mathematical tenets that are learned largely in isolation from other subject content.

Recognizing that children have difficulties discerning between reality and representation, Ajello, Bombi, Pontecorvo, and Zucchermaglio (1987) point out the challenges of conflicts between teacher and expert knowledge. Classrooms could benefit from discussions about these conflicts, enable students to see the contexts for making decisions, and consider the possible means of their reconciliation. Bringing various community representatives into the classroom offers further information for student consideration (Crain, Goodwin, Herd, Ragan, & Ragan, 2006; Greenspan, 2005).

If elementary teachers experience discomfort with teaching both mathe-

matics and personal finance tenets, they lack the ability to determine the accuracy and appropriateness of their curriculum materials. Hamburg (2009) notes that may teachers lack understanding of many financial concepts. This ignorance prompts a dependency upon textbooks to support classroom discussion. Additionally, these teachers who lack financial knowledge may depend on financial education materials of dubious content integrity or commercial bias from outside sources (Stanger, 1997). Silver et al.'s (2009) finding of low level mathematical expectations among elementary teachers indicates that these teachers may possess limited abilities to critically evaluate such materials. Preparing elementary teachers to teach personal finance and associated mathematical tenets necessitates that teacher educators challenge candidates' preconceptions towards content associated with mathematics and personal finance and facilitate authentic experiences for application of understandings.

Pedagogy

While teachers should know the nature of the content prior to teaching it, they also should create the most efficient processes to enable student learning. For example, to ensure convergent thinking about content, teachers may employ direct instruction for the teaching of concrete facts or skills. On the other hand, teachers may facilitate discovery of open-ended content through cooperative, student-centered experiences to prompt divergent thinking. Similar conditions affect elementary and middle school prospective and practicing teachers. To strengthen their lack of knowledge about the links between financial formulas and mathematical processes, preservice and practicing teachers should explore the mathematical connections using available tools such as the computer in simulated context.

Oakes and Lipton's (2007) description of the historical struggle between advocates for traditional and progressive instructional mathematics relates to this situation. How is it possible for students to develop contextually relevant understandings of personal finance through instruction that emphasizes calculation procedures? When teacher candidates lack the mathematical understandings to convey this information to their prospective students, they unwittingly continue a cycle of mathematical ignorance that challenges understandings of mathematical formulas that are essential to understanding income issues in financial education. For example, the use of coupons is commonly touted as a method for discounting price; however, if the coupon does not relate to an item that is considered within a spending plan, the discount is irrelevant. Similarly, there are consequences for using the improper types of spending tools. Middle level students should examine outcomes of cash-based transactions and relate them to the potentially exponential consequences of credit-based spending. Unless teacher candidates can communicate practical skills, such as reducing impulse buying

and delaying gratification, absent model domestic settings, their students stand at risk of being unable to intelligibly discuss these matters.

Recommendations

Financial education processes are inadequate for children in grades K-8. This paper presented the conditions that prompt these environments. Future policy efforts should address these challenges to foster equitable learning processes. To that end, we provide the following recommendations.

Legislative. Development and implementation of mathematically and socially integrated financial education learning standards for grades K-8 are advocated. Such standards should be flexible to enable the success of children who experience different financial contexts, including socioeconomic. Establishing practical expectations for content learning and providing reinforcement of material may bring about learner mastery with the material.

Curricular. Financial education represents a multifaceted concept that affects several different content areas. Since curricula should parallel the nature of the content, curricula of several content areas, including mathematics, social studies, and language arts, should address financial education tenets, thus providing students with the reinforcement to enhance subject learning. For example, the educators could meet standard "Make financial decisions by systematically considering alternatives and consequences" (Jump$tart Coalition, 2007, p. 9) through a social studies lesson that examines patterns in different peoples' needs and wants, a mathematics lesson that compares the cost per unit of each item, and a reading lesson about the financial planning decisions that children make. Curricular efforts should build upon the needs of students and their communities. Financial planners employ a basic principle of matching financial products to the financial profiles of their clients. Similarly, financial education curricula must match the financial education needs of the learners. The following activity examples in Table 1 show the connection among the content areas of mathematics, social studies, and personal finance.

By realizing the benefits of self- or community- developed education materials that address the financial topics of most interest to learners and creating them, education settings may teach those topics most pertinent to their children's development. Teachers and administrators should be cognizant of the difficulties associated with employment of commercially produced curriculum materials and difficulties of one-size-fits-all curricula.

Instruction. The use of inquiry and discovery learning to enable children's construction of financial education content and the associated life connections is encouraged. Pedagogies based on memorization of values and mathematical formulas contradict the natural processes of child devel-

Table 1. Example K-8 activities that connect Mathematics and Personal Finance

Gr.	Mathematics Topics	Financial Literacy Expectations
K	Sorting through differences	Compare prices for the same item at two different stores.
1	Value of coins as related to other US coins	Prepare a personal property inventory, assess value in general.
2	Consumer buying	Explain how peer pressure can affect spending decisions.
3	Estimating strategies for determining sums of money	Explain the difference between a career and a job and identify jobs in area.
4	Allowance and Spending Plans	List examples of financial decisions and their possible consequences.
5	Comparison Shopping	Given an age-appropriate scenario, describe how to use systematic decision making to choose among courses of action that include a range of spending and non-spending alternatives.
6	Credit and Debts: Choosing Wisely	Discuss the advantages and disadvantages of different payment methods, such as stored value cards, debit cards, and online payment systems.
7	Earning & Saving: Income and Careers	Compare personal skills and interests to various career options.
8	Determining the appropriate insurance coverage.	Explain how insurance deductibles work.

opment and impair their future abilities to think critically about the complex financial realities of the early 21st century. Because financial education represents a multidimensional area, that involves mathematics, social studies, and literacy, children need exposure to "hands-on" experiences that encourage interactions with peers to enable the social connections associated with his citizenship area. To help demonstrate how mathematical problem solving pedagogical approaches can enhance a student's learning through patterns and generalizations, Table 2 offers ideas for learning activities. Also see the Appendix for specific problems and activities.

These illustrations of sample mathematical problems that can be incorporated into the specific grade levels will help the reader to clarify the intent of the authors about how the joint blending of mathematical topics and literacy tenants may be utilized in the K-8 classrooms. The teacher educators can also incorporate these problems into methods courses for preservice teachers.

Teacher preparation. Primary, elementary, and middle school teacher ed-

Table 2. Specific and detailed math topics and sample problems that relate to financial literacy

Grade Level	Financial Literacy/ Mathematics Topic	Financial Literacy Sample Problem Listed numbers are example questions that follow this Table
Kindergarten	More, Fewer, Same	Place a number of items on the overhead projector. Have the student place "more than;" "less than," "the same number" on their desk. Make a game out of this! (#1)
	Sorting through differences	Tell the differences between coins: Penny, nickel, dime, quarter, using both "heads" and "tails" of the coin. (#2)
	Making money decision	See http://www.practicalmoneyskills.com for ideas of what is free versus items to buy, noting varying costs.
Grade 1	Beginning recognition of US coins	*One Dollar: My First Money Book* by Lynette Long. A look at money and its value. Create a lesson plan including the discussion of this book.
	Value of coins as related to other US coins	Using a specific coin, find other coins that are equivalent to the same value. For example, one dime = 2 nickels, or 1 nickel and 5 pennies.
	Saving of Money	Create jobs within the classroom, where bonus money can be earned.
Grade 2	Recognizing values of US coins	Students are given pictures, both "head" and "tails" of U.S. coins. They are asked to record on a chart how many of each type of coin they have. (#3)
	Grouping coins for more efficient counting	Using a bag of coins they are given, show at least 3 different ways to show 20¢, 25¢, and 50¢. (#4)
	Consumer buying	Set up a "grocery store" within the classroom. Have groups of students plan purchases of a number of items, with a purpose in mind (healthy food, quantity buying, etc.). Keep a chart of the cost of foods (addition and multiplication tasks).
Grade 3	Estimating strategies for estimating sums of money	Give student scenario of buying items. See (#5) for decision-making tasks. This scenario involves estimation tactics in real life situations.
	Ways to obtain fewer coins as change when buying items	Give student various situational experiences where a limited number of coins are desired. (#5 and #6)
	Community awareness and assistance	Find a local cause. Brainstorm ways to gain financial resources to financially contribute to this cause. Seek assistance from family, friends, and community.
Grade 4	Money Responsibility Allowance Chart with decisions	*The Kid's Allowance Book* by Amy Nathan provides a basis for creating various scenarios in a family. Follow this family for a month.
	Saving & Investing	http://pbskids.org/cyberchase/ and http://www.younginvestor.com/
	Allowance and Spending Plans	Using the Practical Money Skills from *Money Math: Lessons for Life* activity give students a set amount of beans representing "money." Given the scenario of categories, have each student develop a spending plan when a fixed income occurs. Have each student create a graph indicating their selection of spending, followed by a detailed reason for their decisions.
Grade 5	Salary chart	Divide a set amount of money (beans) into categories of "spend," "save," and "give." Develop a plan for use of "money" during a month-long simulation activity where a set allowance is given, expenses levied, and life situations occur.
	Budging: Your financial resources	See Lesson 1 in http://www.practicalmoneyskills.com/foreducators/lesson_plans/children.php
	Comparison Shopping	Students in groups of 4 will be asked to pick a popular item, such as soft drinks. Each group will investigate the costs of several types, calculate the unit price, and create a chart of comparing attributes. Discuss choices as related to money expenditures.
Grade 6	Financial Responsibility & Decision Making	See *Money Math: Lessons for Life* by Suiter and McCorkle (2008) see http://www.treasurydirect.gov/tools/tools_moneymath.htm the Budget to download the curriculum.
	Credit and Debts: Choosing Wisely	Create a scenario dependent on the predominant economic status of the students in the class, whereby each student is given a series of situations. Students must decide what to do to mediate their desires with their necessities.
	Math and Taxes: A Pair to Count On.	See *Money Math: Lessons for Life.* See curriculum at http://www.treasurydirect.gov/tools/tools_moneymath.htm Click on PDF file *Math and Taxes.*

Grade 7	Earning & Saving: Income and Careers	Investigate various careers, needed education in combination with career income. Compare and contrast the amounts earned as related to specific years of longevity—10 years, 20 years, 30 years. Determine a savings plan; calculate the amount earned for the same number of years.
	Giving & Owing	The Mint from Northwestern Mutual and the National Council on Economic Education is a good resource. There are interactive activities on earning, saving, spending, tracking, investing, owing, giving and safeguarding. See http://www.themint.org/
	Credit & Debts: Buy now and pay more later	*It All Adds Up*: On-line games on credit, buying a car, budgeting, saving, investing, and paying for college. See http://www.italladdsup.org/ by National Council on Economic Education.
Grade 8	The Stock Market: Risk and Rewards Spending and Investing	See Money Math: Lessons for Life. See curriculum at http://www.treasurydirect.gov/tools/tools_moneymath.htm Click on PDF file The *Secret to Becoming a Millionaire.*
	Risk Management & Insurance	*Get Ready to Take Charge of Your Finances* curriculum found at http://www.doe.in.gov/octe/facs/IndianaFinLitEd-MiddleSch-8WeekUnit.html designed by Family Economics and Financial Education (FEFE) project and piloted by middle school teachers in Indiana.

ucators should include financial education topics as part of their programs. Such endeavors should challenge preconceived understandings of financial topics and related mathematics issues. While such preparations should be included within both mathematics and social studies methods coursework, a comprehensive learning experience cannot occur without a full course in financial education methods for elementary and middle school teachers as part of their advanced (e.g., graduate studies) learning. Such a course should provide enrollees with information about the content, introduce personal finance standards, model constructivist activities, and enable creation of contextually appropriate lessons.

Through these efforts it may be possible to broaden conceptions of mathematics and their connections to financial literacy. Such endeavors may recognize the importance of valuing different content perspectives and approaches to problem-solving that counter deficit views of mathematics learning. One strategy may adapt Lowenstein's (2009) three suggestions (field-based experiences, reflection opportunities, and student involvement) for erasing deficit views of white teacher candidates' perspectives of multicultural education. As it relates to connections between mathematics and financial literacy, we advocate employment of field experiences that require cooperative mathematics instructional strategies that employ problem solving opportunities encouraging for multiple acceptable solutions. Such learning opportunities are necessary for broadening students' conceptions of mathematics and its applications.

In addition, the regular reflection about the broad conceptions of mathematics and their applications is encouraged. Brantlinger (2008) experiences with urban high school students illustrates the problems of trying to facilitate conversations about real world mathematics issues when students lack the basic financial knowledge to make these connections—information that should have been garnered in elementary school settings. Encouraging

reflection about both the concrete and abstract dimensions to mathematics may prompt candidates' realization of the appropriateness of teaching all dimensions.

Finally, student involvement is paramount to the learning process. Garnering the real world interests of students and inviting suggestions for financial topics that they encounter and wish to study provides them with elements of ownership in their learning.

Conclusions

Financial literacy represents a critical mathematics education issue for primary, elementary, and middle level learning. The linear perceptions of personal financial and mathematics represent shallow understandings of these areas that inhibit child development. Fuller understandings of personal finance, in all of its dimensions, are required in a setting that defines identity through economic means (Bobbitt, 2002).

The challenge for mathematics educators represents a daunting, though not insurmountable one. In a standards-focused climate that stresses testing preparations, mathematics educators face the responsibility of examining the interdisciplinary connections to mathematics and encouraging their students' discovery of personal finance as a matter of more than budgets and spreadsheets.

Following Paulsen and St. John's (2002) lead, studies that focus on the students' socioeconomic classes and the associated influencing experiences are encouraged. Lucey (2005) presented evidence that the popular Jump$tart surveys of financial literacy among high school seniors and college students may involve elements of social bias; additional research needs to examine the extent of such bias and the effect on understandings of teachers and students in various contexts.

Additionally, exploration into the nature of multiple teacher preparation programs and their preparation of candidates to teach financial education tenets in elementary and middle grades is recommended. Just as children experience dramatic development changes between kindergarten and eighth grade, elementary and middle grades preservice teachers may possess different degrees of mathematical understandings as indicated by their intended teaching grade. Developmentally appropriate mathematics teaching depends upon teachers possessing knowledge of both the content and learning processes to critically examine curricular materials and their appropriate classroom use. Planning considerations include the different views of these topics brought by students of different contexts and how to devise appropriate instructional techniques. By connecting preservice teachers' mathematical understandings with their planned grade level expertise and

cultural context, it may be possible to prompt equitable learning conditions for children of all socioeconomic contexts.

A comprehensive resource that addresses all content relationships among mathematics, social studies, and personal finance would be remarkable work that would need to be addressed in another paper. Suffice it for the current paper to provide a sample of the interdisciplinary financial literacy concepts. This paper has described deficiencies in teaching and teacher preparation as they relate to mathematical and financial literacy. The issue presents a matter of inconsistent objectives among stakeholders that find children caught in the middle. The solution represents one of practicality. Mathematics educators cannot stand idly by. Collaborative solutions are needed wherein mathematics educators reach out to those associated with other content areas and examine the broader mathematical applications to personal finance.

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Appendix

The following examples are provided to illustrate the various ways of wording and presenting problems to encourage the student to think authentically rather than to just give answers (often numerical) prescribed by a text or instructional guide. Thus, they demonstrate ways of structuring problems that students may encounter in their personal lives because they are more realistic than the ones located in the traditional mathematics textbook problem-solving section. See these problems references in Table 2.

1. Here are some coins. Put some of the coins in the block on the right.
   Now, place some of the same type of coins on the left, such that there are "fewer coins."
   Do the same, except show "more coins."
   Do the same, except show "the same number of coins."

2. In the cup, there are 4 coins. Select any two coins.
   How are the coins the same?
   How are these different?

3. The value of all the coins shown below is $1.
   [One area has 7 coins: front of quarter, back of another quarter, front of a nickel, back of a nickel, three pennies, two with heads one with tails. Another area has three dimes—one head, 2 tails; one nickel, heads, 2 pennies: one heads, one tail]

   1. Fill out the chart below by indicating how many nickels are needed to make 10 cents.
   2. Fill out the chart to indicate how many pennies are needed to make 5 cents.
   3. Fill out the chart to indicate how many quarters are needed to make 50 cents.

Area	Quarters	Dimes	Nickels	Pennies
1.				
2.				

4. You are given a bag of US coins. Find at least 3 different ways to show the following amounts. Draw a picture of the coins you used in the space provided.

Amount	Picture of Answer
20 cents	
25 cents	
50 cents	

5. Here is a grocery advertisement of various items. Estimate the total cost of the items to the nearest dollar _____
(These items should be either very close to the next dollar, (e.g., $2.99) or easily recognized amounts (e.g., $1.25)

Select three of these items under each of the described conditions.

a. You are given $5. Find three items you can select that you will not have enough money to buy.
b. You are given $5. Name three items that you can buy. How much change will you receive if you use the entire amount to purchase all three items?
c. You are given $5. What three items can you buy for this amount and receive the smallest amount of change back? What three items can you buy that will give you the most amount of change back?

6. Why do some people give the clerk $1.02 instead of $1.00 when an item costs 52¢? When are some other times that it would make sense to add change to the bills used to pay for an item?

http://www.ncee.net/personalfinance
http://www.themint.org/
http://www.practicalmoneyskills.com/foreducators/lesson_plans/