COGNITION AND INSTRUCTION, 1989, *6*(4), 287–318

Activation of Real-World Knowledge in the Solution of Word Problems

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This article describes two studies that examine factors influencing children's access to real-world knowledge during the solution of word problems. In the first study, based on work in Brazil by Carraher, Carraher, and Schliemann (1987), children were asked to solve arithmetic problems presented in three contexts: (a) as word problems, (b) in simulated store situations, and (c) as symbolic computations. Brazilian children were both more successful and more likely to use mental, informal strategies when solving word problems than when solving symbolic computations. We did not find the same results with our U.S. sample; no effects of context were found in either strategy use or success. Comparison of U.S. and Brazilian children's responses suggested that children may tend to access real-world content when the numbers in a word problem match the problem content, and a second study was conducted to test this interpretation. Children were presented with word problems in which the problem content either matched or did not match the numbers in the problem. It was found that when the numbers matched the problem content, children were more successful in solving the problems and more likely to access their domain knowledge during problem solution, as evidenced by the strategies they used to solve problems in the matched condition. These findings suggest ways in which activation of real-world knowledge might be facilitated during the solution of word problems in school.

How children transfer knowledge between school and the outside world may be the central problem in education. Most educators would agree that we

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teach children to add, subtract, multiply, and divide in school so that they will be able to apply these skills to solving problems they encounter both inside and outside school. Traditionally, one of the functions of school word-problems has been to allow children the opportunity to apply their skills to real-world problem-solving contexts. It becomes increasingly clear, however, from studies of problem solving in school and at work, not only that children have difficulty applying school mathematics to the solution of word problems in school, but also that school graduates who routinely solve quantitative problems in the course of their work and daily activities do not generally use the mathematical algorithms they were taught in school. We are thus left with a substantial gap separating the mathematical skills taught in school from strategies developed for solving quantitative problems in nonschool contexts. In the two studies reported here, we seek to determine whether strategies employed successfully in one context are also employed in other contexts, and what factors might influence this kind of transfer.

The highly contextual nature of human problem-solving behavior has been demonstrated in several recent studies. Despite the fact that American adults have been subjected to many years of schooling in which they were taught to perform various algorithms for solving quantitative problems, it is well documented that they generally do not use these algorithms for solving problems arising in their jobs and daily lives. As examples, Lave, Murtaugh, and de la Rocha (1984) studied adult grocery shoppers engaged in calculating and comparing costs of various grocery items, and Scribner (1984) studied the procedures used by dairy workers in solving quantitative problems arising in their daily work. These studies found that schooled adults generally do not use school-learned algorithms when solving quantitative problems embedded in daily routines. Rather, they use special-purpose strategies constructed within the specific problem-solving context.

Just as school-taught algorithms do not easily make their way into real-world problem-solving contexts, so too, aspects of real-world situations do not easily make their way into the classroom. Even though one educational rationale for assigning word problems in school is to give children practice in solving problems that might be encountered in nonschool settings, there is ample evidence that many children do not bring their knowledge of the world to bear on solving word problems. Rather than functioning as contexts, word problems, by middle elementary school, function as symbolic puzzles that are perceived as being separate from the real world. Word problems confuse children, causing them to suspend their otherwise good judgment and behave in irrational ways, zeroing in on key words or anything else that will allow them to dispense with the analysis and begin computing (Reusser, 1986; Schoenfeld, 1988).

Studies of problem solving in context are extremely valuable. They enable us to understand the way situations are represented by the human cognitive system and to see the impact these situation-specific representations inevitably have on problem solving. There is no doubt that human problem-solving ability is grounded on highly contextual knowledge representations, not on collections of generally applicable algorithms. Yet at the same time, individuals function across many contexts and often activate in one context knowledge gained in another. Although school-taught mathematical procedures are seldom applied in everyday life, this does not mean that such knowledge transfer is impossible. Indeed, formal mathematics in particular has been applied across a wide range of contexts (cf. Paulos, 1988), albeit mostly by specialists. But if mathematicians can apply formal mathematics to real-world situations, why not other people as well?

We accept the view that knowledge is highly domain-specific, and the work on problem solving in everyday contexts strongly supports this view. We seek to investigate, however, how it can be, given this domain specificity, that knowledge ever transfers from one context to another? In particular we focus on word problems of the type presented in school. What conditions might enable children to access knowledge gained outside school while engaged in solving word problems?

Given the fact that children in the United States are known to have a great deal of difficulty in solving word problems, a recent study in Brazil had particular relevance to our research question. Carraher, Carraher, and Schliemann (1987), working in Brazil, set out to show that children's quantitative knowledge is so contextually specific that, even within the context of school, different kinds of problems elicit different problem-solving strategies. Carraher et al. compared children's problem-solving strategies on the same computational problems presented in different contexts. The same authors had shown in a previous study (Carraher, Carraher, & Schliemann, 1985) that unschooled children working as market vendors solved quantitative problems more easily when the problems were embedded in a market context than when the problems were embedded in a school-like context. In their 1987 study, Carraher et al. hypothesized that schoolchildren might similarly vary their solution strategies according to the contexts in which problems were presented. They tested this hypothesis by observing a group of third graders from a poor area in Brazil solving the same set of problems embedded in three different contexts: (a) symbolic computation exercises, (b) standard word problems, and (c) a simulated store situation in which the experimenter took the role of the customer and the child that of the sales clerk.

The researchers found significant differences in success rates across contexts: Children were much more successful in solving word and store problems than in solving computation exercises. Furthermore, the children were more likely to use mental computation strategies in solving word and store problems and to use written strategies in solving the computation exercises. The use of mental strategies was associated with a greater probability of success across all operations and contexts, and protocol analyses revealed that the use of mental computation strategies in general was tied to a manipulation-of-quantities approach to problem solving, characterized by the decomposition of large numbers to produce small, easily manipulated subtotals.

Carraher et al. interpreted their results as indicating that the context in which a problem is presented influences selection of a solution strategy. They postulated that children were more successful in solving word or store problems because the quantities were embedded in meaningful transactions. By infusing the problems with meaning, it is possible to elicit informal, manipulation-of-quantity solution strategies. If children are more successful in applying informal strategies than in applying the more formal algorithmic strategies, then embedding problems in meaningful contexts leads to greater success.

We were interested in the results of the Brazilian study because of the way in which Brazilian children seemed able to use knowledge gained in other contexts to help them solve word problems in school. Although the Brazilian children treated word problems as a kind of "real-life" context and, therefore, found them easier to solve than computation exercises, this is apparently not the way U.S. children treat word problems. It is well known that U.S. children find word problems quite difficult and generally are as successful, if not more successful, in solving a symbolic computation problem as they are in solving a word problem in which the same computation is embedded, particularly if the word problem is a complex one (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). U.S. children do not necessarily treat word problems as meaningful contexts and tend not to relate those problems to what they know about problem solving out of school. That Brazilian children were more proficient at solving word problems than they were at solving symbolic computation problems is a fascinating result, and it runs counter to results found for U.S. children. Perhaps by looking more closely at the Carraher et al. experiment we can obtain some clues concerning why, in this case, children were able to activate real-world knowledge in a school task.

This article describes two studies that investigate the relationship between problem context and strategy choice in children's solution of quantitative problems. The first study uses the experimental paradigm of the study done in Brazil by Carraher et al. (1987) to examine the behavior of a group of U.S. children given the same problems. The second study investigates in further detail the relationship between problem context and strategy choice, based on results of the first study.

STUDY 1: A COMPARISON OF U.S. AND BRAZILIAN CHILDREN'S RESPONSES TO THE EXPERIMENTAL PARADIGM USED BY CARRAHER ET AL. (1987)

Carraher et al. began their work with a careful ethnographic description of the problem-solving contexts and strategies that could be observed among

Brazilian children of working-class families (Carraher et al., 1985). They then constructed their experimental study to validate their ethnographic findings. The goal of our study was quite different. Had we wanted to replicate the Carraher et al. study, we would have started by doing ethnographic work with U.S. children. Instead, we simply wanted to follow up on one puzzling aspect of Carraher et al.'s results, namely, that Brazilian children treated word problems as contexts, whereas U.S. children apparently do not. In this first study then, our goal was simply to explore U.S. children's responses to the particular experimental procedure that Carraher et al. had used with Brazilian children. By comparing the results, we hoped to find clues that might help us understand differences between these two populations.

Method

Children. Eighteen U.S. children, all of whom had completed third grade and not yet begun fourth grade, participated in the study. They were all from large urban areas, but the type of school they attended varied. Thirteen children attended either public or private schools in Chicago, 3 attended suburban public schools, and 2 children attended school in other large cities.[1]

Procedure. The design of this study followed almost exactly that of the Brazilian study. Three problem sets of 10 problems each were presented across three contexts: symbolic computation, word problems, and store problems. Each child received all three problem sets (a total of 30 problems) and solved each set in one of the three contexts. The problems and contexts were identical to those used in the Brazilian study, except that names of monetary units were changed to U.S. ones. Table 1 shows the problem sets as they appeared in the three contexts: as symbolic computations, word problems, and store problems.

Certain features of the design of the Brazilian study seemed problematic. Some of these features were changed in our study, and some were retained but acknowledged as problematic and analyzed accordingly. In both the Brazilian study and in our study, the three problem sets were always presented in the same order. Assignment of the three sets to the three contexts and, thus, the order of contexts were varied across participants. Although Carraher et al. varied presentation of the three contexts according to a Latin-square design, we used a factorial design, yielding six possible orders for presenting the three contexts. The factorial design allowed us to counterbalance completely across context orders, which was not possible using a

[1]The children who participated in the Brazilian study were in third grade and attended a public school in an impoverished urban area.

TABLE 1
Problems Used in the Replication of Carraher, Carraher, and Schliemann (1987)

Problem sets

I. 60 + 240	115 + 15	195 + 57	40 x 3	12 x 50
200 − 35	210 − 105	143 − 68	100 ÷ 4	75 ÷ 5
II. 420 + 80	115 + 15	195 + 57	4 x 25	15 x 50
500 − 70	210 − 105	252 − 57	100 ÷ 4	120 ÷ 3
III. 80 + 240	185 + 68	106 + 106	3 x 40	50 x 12
200 − 35	210 − 105	243 − 75	100 ÷ 4	75 ÷ 5

Word problems

1. John had __ marbles. He played with Paul and won __ marbles. How many marbles does he have now?
2. Jack bought a ball for $__ and a car for $__. How much money did he spend altogether?
3. Mark went to see a movie. He spent $__ for the bus and $__ for the movie. How much did he spend altogether?
4. I bought an orange for $__. I paid with $__. How much change did I get back?
5. Robert had __ marbles. He played with a friend and lost __. How many does he have now?
6. I had __ baseball cards in my collection. I lost __. How many do I have now?
7. In a school there are __ classrooms. In each classroom there are __ children. How many children are there in this school?
8. Peter bought __ eggs. Each egg costs $__. How much money did he spend?
9. Mr. Roger gave __ marbles to __ children to share amongst themselves. Each one should get the same amount as the others. How many marbles did each child get?
10. Marie gave $__ to __ children who washed her car. They divided the money so that each child had the same amount as the others. How much money did each child get?

Store problems[a]

1. One ring costs $__. One large candy bar costs $__. I want one of each. How much do I have to pay?
2. You sold __ pencils yesterday and __ today. How many did you sell altogether?
3. Let's say this doll costs $__ and this pencil costs $__. I'm buying both. How much do I have to pay?
4. I want to buy this pen that costs $__. I'm paying with $__. How much change will you give me?
5. I have $__ in my pocket. I want to buy this bag of marbles with it. You're selling the bag for $__. How much money will I have left?
6. The pen costs $__. I'm paying you with $__. How much is my change?
7. You're selling each of these pencils with erasers for $__. I want __ of them. How much will I have to pay?
8. Each pencil costs $__. I want __ pencils. How much do I have to pay?
9. __ of these squirt guns cost $__. How much will you sell one of them for?
10. You're selling __ cars for $__. I only want one. How much does one car cost?

[a]The original set of store problems in the Carraher et al. study contained four subtraction and two addition problems. We chose to include equal numbers of addition and subtraction problems to make the store context comparable to the other contexts. Thus, Store Problem 2 in the U.S. study replaces the original problem reported by Carraher et al., "You had __ pencils. You sold __. How many do you still have in your store?"

Latin-square design. Three children were randomly assigned to each of the six context-order cells. Preliminary analyses showed no significant effects of context order, so cell divisions were collapsed for subsequent analyses.

A more serious problem with the Brazilian study was that some of the problems appeared in more than one problem set and, thus, were solved by the same child more than once, albeit in different contexts. An examination of the problem sets revealed that four problems were repeated in two different problem sets (115 + 15, 195 + 57, 200 − 35, 75 ÷ 5), and two problems were repeated in all three sets (210 − 105, 100 ÷ 4). Because Carraher et al. did not mention these repeated problems in reporting the results, it was not clear how these problems would affect the results of our study. Because we wanted to follow Carraher et al.'s procedures as closely as possible in this study, the problem sets were not altered to replace repeated problems with novel ones. As we show later, however, some of these problems played a central role in the present study's findings.

In both studies, each child participated in one problem-solving session in which a total of 30 problems was presented, 10 in each of the three contexts. The experimenter presented the problems orally to each child and instructed the child to solve the problems whatever way was easiest, indicating that problem solutions carried out mentally or using pencil and paper were equally acceptable.

In the store context, all items mentioned in the problems were laid out in front of the child, who was free to use them as manipulative aids in solving the problems. In our study, most children did not do so. If a child's method for arriving at a solution was unclear, the experimenter asked the child to explain the procedure used, continuing with neutral probes until the experimenter understood the nature of the strategy used. All sessions were tape-recorded.

Coding. Each child's answer to each problem was coded as either *correct* or *incorrect* and according to the strategy used to arrive at the answer. Following Carraher et al., strategies were coded as either *oral* or *written*. When children solved problems entirely in their heads (i.e., without using pencil and paper), the strategy was coded as *oral* (following Carraher et al., and henceforth referred to as *mental*). Strategies were coded as *written* when pencil and paper were used to arrive at the solution.

We found it necessary to code two additional strategies. Although Carraher et al. did not report the percentage of children who refused to answer a problem, the U.S. children often refused to give an answer. When this happened, the problem solution was coded as *incorrect*, and the strategy was coded as *skipped*. A fourth strategy coded was that of remembered problems. Again, Carraher et al. did not report whether any Brazilian children remembered their previous encounters with problems that were repeated, but it was found that many U.S. children remembered at least one problem. For exam-

TABLE 2
Mean Number of Problems Solved Correctly of a Possible 10 in Each Context Compared With Results From Carraher, Carraher, and Schliemann (1987)

Sample	*Computation*	*Word*	*Store*
Brazilian	3.8	5.6	5.7
U.S.	7.0	7.0	6.9

ple, a child might recognize a problem as one already encountered, say "I've already done that," and proceed to search the scratch paper for the previous solution. These problems were coded as *remembered*, and they were scored correct or incorrect appropriately.

Results

We begin our presentation of results by describing the analyses that replicate those done by Carraher et al. and comparing the two sets of results. Then we turn to additional analyses carried out using only the U.S. sample.

Replication Analyses

Number correct. Summary variables were constructed for each child indicating the total number of problems solved correctly in each of the three contexts. These summary scores were then analyzed using a repeated-measures analysis of variance (ANOVA), where context was treated as a within-subject variable with three levels. Table 2 shows the mean number of problems solved correctly in each context, from a possible 10 problems per context, for the U.S. and Brazilian samples. Whereas Brazilian children were found to be more successful in both the word problem and store contexts than they were in the symbolic computation context, these results were not replicated with the U.S. children, who displayed no significant effect of context whatever, $F(2, 34) = .05$, $p = .95$. It is interesting to note that not only was there no context effect for the U.S. sample, but the U.S. children also solved more problems correctly in every context than did the Brazilian children.[2]

Strategy choice. Carraher et al. reported significant differences in strategy choice across contexts for the Brazilian children, with mental strategies more common in the word and store contexts, and written strategies more common in the symbolic computation context. To test whether there were any effects of context on strategy choice in the U.S. sample, each child

[2]The lack of context effects was not due to ceiling effects in the U.S. sample. Only 2 of the 18 U.S. children solved all problems correctly, and the median number correct was the same as the mean number correct, a finding inconsistent with the skewed distribution that would be associated with a ceiling effect.

was given two new summary scores reflecting the total number of problems out of 10 in each context that he or she attempted to solve using mental or written strategies. Each of these new summary scores was analyzed separately as described earlier, using repeated-measures ANOVAs. Again in contrast to the Brazilian findings, no differences were found across contexts in the tendency to use either mental, $F(2, 34) = 0.80$, $p = .46$, or written, $F(2, 34) = 1.19$, $p = .32$, strategies.

Table 3 shows the mean number of problems attempted using mental and written strategies for the U.S. sample. Although Carraher et al. did not report the mean number of strategy attempts separately for each context, they stated that the average number of problems solved using written strategies across all contexts was 13.6 out of a possible 30. This is much lower than the U.S. average of 18.9 ($SD = 5.71$). So, although the U.S. children outperformed the Brazilian children in all contexts, they were less likely overall to use mental calculation strategies. Furthermore, whereas Carraher et al. reported a significant relationship between strategy choice and context, with word and store problems eliciting more oral strategies, strategy choice was independent of context in the U.S. sample. For U.S. children, particular problem contexts did not elicit any particular strategies.

Additional Analyses

Strategies and success. Carraher et al. found that, regardless of context, mental strategies were associated with a greater probability of getting the correct answer. They interpreted this to mean that using a mental strategy helps the problem solver answer the problem more easily. We have already shown that the U.S. children got more problems correct on average than the Brazilian children and used a higher proportion of written strategies. We were still curious to know, however, whether children were more likely to obtain a correct solution when attempting to solve a problem using mental strategies (as in the Brazilian sample) or using written strategies.

To ascertain whether an association existed between U.S. children's use of mental strategies and their success in solving the problems, correlations were computed across individuals between the total number correct and the proportion of written and mental strategies used. There was a small positive correlation between the total number correct and proportion of mental

TABLE 3
Mean Number of Problems Attempted in Each Context Using Different Strategies

Strategy	*Computation*	*Word*	*Store*
Mental	2.20	2.70	2.70
Written	6.70	6.20	6.10
Remembered	0.39	0.39	0.39
Skipped	0.72	0.78	0.78

strategies used ($r = .406$), and a small negative correlation between the total number correct and the proportion of written strategies used ($r = -.346$). It seems that, within the U.S. sample, individuals who used a higher proportion of mental strategies also tended to obtain a larger proportion of correct answers than did individuals who used more written strategies, a finding similar to that in the Brazilian study.

These data do not allow us to infer, however, that using mental strategies causes an increase in the likelihood of obtaining correct answers. It is equally plausible, in fact, that children who are better at solving problems in general may tend to use more mental strategies, but that use of mental strategies may not necessarily cause any given student to get more problems correct. If mental strategies cause correct answers, we would expect that, within individual children, attempts in which mental strategies are used should be more likely to result in the correct answer than attempts in which written strategies are used. To test this prediction, we calculated for each child the proportion of attempts using each strategy—mental and written—that resulted in the correct answer. Of the 18 children in our sample, 9 got a greater proportion of mental attempts correct (as opposed to written), 7 got a greater proportion of written attempts correct, and 2 children got all attempts correct. The theory that the mental strategies necessarily lead to greater success in arithmetic problem solving is thus not supported by our data. It is more likely that children who are better at solving quantitative problems use mental strategies more often than children who are less successful problem solvers. That is, if all children use mental strategies on easier problems, then for a given set of problems, relatively better problem solvers will use mental strategies more.

Operations. In each of the three problem contexts, children solved three addition, three subtraction, two multiplication, and two division problems. Although there were no overall effects of context, we evaluated whether problem context might affect children's ability to correctly solve problems involving particular operations. Problems were divided into addition, subtraction, multiplication, and division, and separate summary scores were derived for each child within each context and operation. Because the number of problems differed across operations, summary scores were computed as proportion correct for problems of each operation within each context. These scores were analyzed using a two-way repeated-measures ANOVA with context and operation as within-subject variables. This analysis revealed a significant main effect of operation, $F(3, 51) = 7.83, p < .001$, and a significant interaction between context and operation, $F(6, 102) = 3.72$, $p < .01$. The mean proportions correct by context and operation are presented in Figure 1.

In general, Figure 1 reveals that children did better on addition problems than on other problems. Scheffé contrasts revealed that children did signifi-

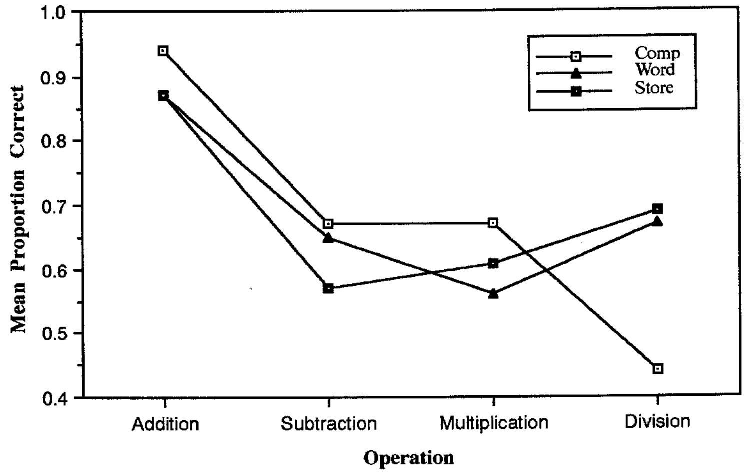

FIGURE 1 Mean proportion of problems solved correctly, broken down by context and operation.

cantly better on addition than on subtraction, multiplication, and division, $F(3, 51) = 4.64$, 5.33, and 5.68, respectively, all $ps < .01$. Performances on subtraction, multiplication, and division did not differ significantly from each other. These results are not surprising given the relative amount of attention these operations had received in the children's school curriculum up to the time of testing.

Far more interesting is the basis for the significant Context × Operation interaction. Simple-effects tests (Winer, 1971, pp. 529–532) revealed that children did better on division problems in the word and store contexts than in the computation contexts, $F(2, 34) = 5.77, p < .01$. There were no significant context effects for the other three operations. Thus, U.S. children's performance was similar to that of Brazilian children for division only. Given this similarity to the Brazilian results in terms of number of problems correct, might the U.S. children's choices of strategies for solving division problems also be similar to the Brazilian children's?

The mean proportions of problems on which mental, written, skipped, and remembered strategies were used are presented in Figure 2, broken down by operation. The pattern of strategy use is similar for addition, subtraction, and multiplication: Children solved roughly three quarters of these problems using written strategies and one quarter using mental strategies. The pattern for division, however, shows an opposite trend, with children solving a higher proportion of division problems using mental strategies than written strategies.

This apparent difference in strategy use between division and the other three operations was tested in a repeated-measures ANOVA with operation as a within-subject variable and the difference in mean proportion of mental

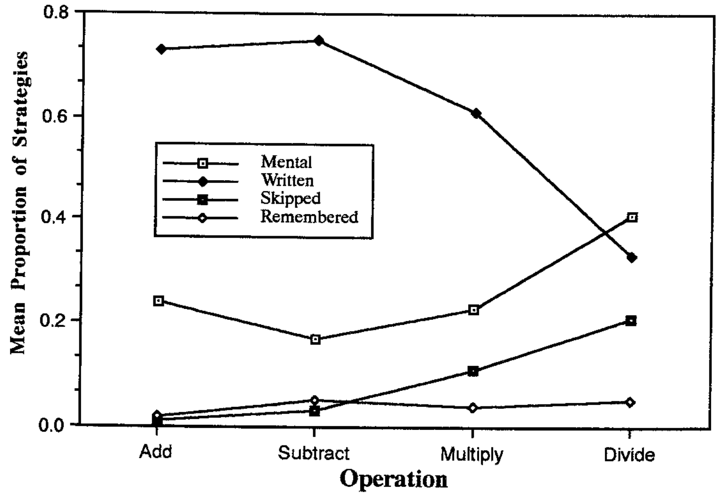

FIGURE 2 Mean proportion of problems solved using different strategies, broken down by operation.

versus written strategies as the dependent variable. Results showed a significant effect of operation, $F(3, 51) = 11.09$, $p < .001$, and post hoc analyses revealed that this effect was due to strategy choice in division differing significantly from that in the other three operations.

Only for division does the U.S. sample follow the pattern described by Carraher et al. (1987), with a greater proportion of mental strategies being connected with more correct answers in the word and store problem contexts than in the computation context.

The case of 100 ÷ 4. To begin to understand why the children were treating division problems differently from problems in the other three operations, we examined the individual problems more closely. It was immediately apparent that one of the division problems, 100 ÷ 4, could be exerting a strong influence on the results. This problem appeared three times; each child solved it once as a computation problem, once as word problem, and once in the store context. As a computation problem, it was presented simply as 100 ÷ 4, but in the word and store contexts, it was presented as $1.00 divided into 4 equal parts. The fact that 100 ÷ 4 was presented as a money problem probably gave children an advantage in the word and store contexts. Most of the children were familiar enough with the U.S. monetary system to mentally divide a dollar into four quarters. Evidence for this comes from the children's explanations of how they arrived at the answer. Most children who solved this problem using mental strategies gave explanations in terms of money, for example, "I count my money every day, so I know that four quarters equals a dollar" or "I play video games and if you play four games that cost 25 cents, you use a dollar." Clearly, most children solved this problem mentally; because it could be so easily mapped onto money, they were likely to get it correct.

We began to suspect that much of the difference in success rate and strategy use between division and the other operations might be due to this particular problem. To test this suspicion, the analyses of proportions correct by context and operation were recalculated, excluding the problem 100 ÷ 4. Although the significant main effect for operation remained, as we expected, $F(3, 51) = 8.31$, $p < .001$, the significant Context × Operation interaction disappeared, $F(6, 102) = 1.38$, $p = .23$.

Similarly, when the problem 100 ÷ 4 was omitted from analyses of the relationship between strategy choice and operation, we found that the pattern of strategy use for division matched that of the other operations (see Figure 3, especially in comparison with Figure 2). A repeated-measures ANOVA on the difference in mean proportion of mental versus written strategies used, omitting 100 ÷ 4, revealed that the previously significant effect of operation had disappeared, $F(3, 51) = 1.10$, $p = .36$. In other words, when the problem 100 ÷ 4 was omitted from the analysis, the mean proportions of mental and written strategies attempted for division were equivalent to those for the other operations.

The pattern of strategy use across contexts, broken down by operation both including and excluding 100 ÷ 4, is presented in Table 4. Again we see that division looks different from the other three operations, but only when 100 ÷ 4 is included. Whereas for addition, subtraction, and multiplication, children solved more problems using written strategies than mental strate-

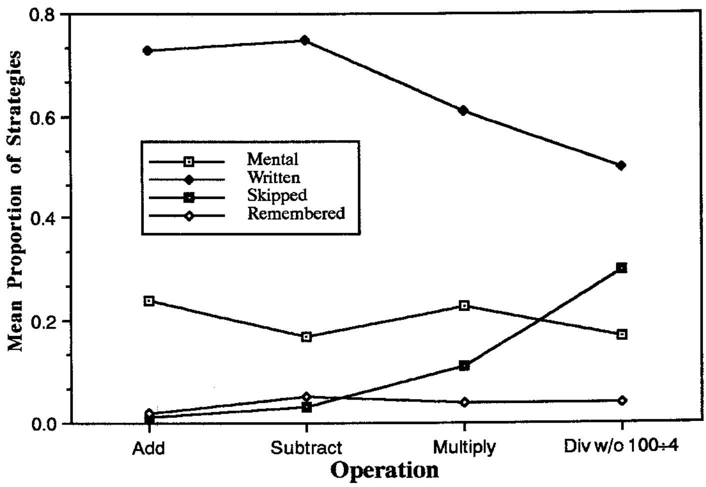

FIGURE 3 Mean proportion of problems solved using different strategies, broken down by operation, excluding the problem 100 ÷ 4.

TABLE 4
Mean Proportion of Problems Attempted Using Different Strategies, Broken Down by Operation

	Operation				
Context and Strategy	*Add*	*Subtract*	*Multiply*	*Divide*	*Divide, Excluding 100 ÷ 4*
Computation					
Mental	.24	.15	.19	.31	.17
Written	.76	.74	.64	.47	.61
Remembered	.00	.09	.05	.00	.00
Skipped	.00	.02	.11	.22	.22
Word					
Mental	.26	.17	.30	.39	.22
Written	.70	.79	.53	.31	.39
Remembered	.04	.02	.03	.08	.06
Skipped	.00	.02	.14	.22	.33
Store					
Mental	.22	.19	.22	.53	.17
Written	.72	.70	.67	.22	.44
Remembered	.04	.04	.03	.06	.06
Skipped	.02	.06	.08	.19	.33

gies in every context, this was true for division problems only in the symbolic computation context. In the store and word problem contexts, children used more mental strategies than written strategies when solving division problems. The omission of 100 ÷ 4, however, causes the division results to match those of the other operations: There is a larger proportion of written strategies than mental strategies, and the proportion of skipped problems increases from addition to division. It is clear that the single problem, 100 ÷ 4, was responsible for the overall context effects in division.[3]

The problem 100 ÷ 4 was analyzed separately, and it was found that although a third of the children attempted to solve the problem using written strategies when it was presented as a computation exercise, not a single child attempted to solve it using a written strategy when it was presented in the store context. Furthermore, whereas 22% of the children chose to skip the problem when it was presented in the computation context, not one child skipped the problem when it was presented in the store context. The case of

[3]The rather high proportion of mental to written strategies for multiplication in the word problem context is due to the problem 25 x 4, another problem with monetary significance. An analysis was done for multiplication similar to that done for division, removing 25 x 4, and similar results were found with respect to mental and written strategies; the proportion of mental strategies used dropped to the level of the other operations and contexts. Protocol analysis revealed children mapping this problem onto the monetary system in the store and word problem contexts. It is unclear why this problem affected strategy choice only in the word problem context and not in the store problem context. The problem was presented only once per session, however, not enough to have any real effect on the results.

$100 \div 4$ suggests that, in addition to main effects of context, there also exist interactions of context with the specific numbers used in problems. It is clear that the children in this study treated $100 \div 4$ as a different problem when it was embedded in different contexts. Because the numbers have a monetary significance, they elicited different strategies when presented in a money context, a context that matches the numbers, than when embedded in a purely symbolic computational context.

There are two approaches to interpreting these results. On the one hand, the fact that the particular problem $100 \div 4$ elicited idiosyncratic problem-solving behavior and that it was included in the study three times could be seen primarily as methodological problems, having threatened the generality of our findings. On the other hand, understanding why this problem elicits responses different from those for the other problems could lead to new insights about the relationship between context and strategy choice. Because U.S. children's responses to $100 \div 4$ resembled Brazilian children's responses to most of the problems, we chose to use the problem as a clue in our quest to understand differences between our study and the Brazilian study. Such considerations will lead directly to the design of Study 2.

Discussion

The responses of U.S. children to the problems used by Carraher et al. were quite different from those of Brazilian children. U.S. Children performed better than Brazilian children in all contexts and used a higher proportion of written strategies than their Brazilian counterparts. Thus, although in Brazil success may be associated with a higher proportion of mental strategies, success across the two samples is associated with a higher proportion of written strategies. Furthermore, although mental strategies were shown to be used more frequently by U.S. children who were more successful problem solvers overall, mental strategies did not lead to more correct problem solutions when analyzed at the individual problem level.

Like the Brazilian children, the U.S. children used a variety of strategies in solving the problems. However, the division of strategies into mental or written misses important distinctions. Informal idiosyncratic strategies may be written, as in the case of the child who solved the problem 12×50 by adding 50 twelve times on paper. Conversely, formal school-taught algorithms may be performed mentally, as did the child who reported multiplying 50×15 in his head by multiplying 5×0, then 5×5, and so on, working just as he was taught to do with pencil and paper in school. The distinction between mental and written is thus of limited usefulness because the same kinds of strategies can be employed in either medium.

The partitioning of strategies into formal and informal, generally meant to capture the same kind of distinction implied by the division into mental and written, is likewise an inadequate concept. Formal strategies are considered to be the algorithms a child learns in school, the manipulation of written symbols with little reference to concrete quantities. Informal strategies, on the other hand, have been characterized as a "manipulation of quantities," a mental approach to problem solving (Carraher et al., 1987; Hiebert, 1984). But this distinction does not capture the differences between decomposing a problem into smaller subproblems, mapping a problem onto a related cultural system of quantification, retrieving number facts, estimating an answer, or counting tally marks. All these would be considered informal strategies, but they seem to be qualitatively different.

U.S. children do not seem to use problem context as a cue in choosing strategies, except when the numbers themselves gain special meaning in relation to the context, as in the case of 100 ÷ 4. The question of why children choose particular strategies requires further attention. We have addressed the issue most directly for the specific problem, 100 ÷ 4. To ascertain why the U.S. study replicated the Brazilian study for this problem only, factors other than general problem context must be considered.

First, although the children in this study had little, if any, experience with written division algorithms, they did have well-functioning addition and subtraction algorithms. It may be that U.S. children's knowledge of division is similar to Brazilian children's knowledge of all formal algorithms, that is, neither well mastered nor well understood. In division only then, the level of problem-solving skills of the U.S. children may have been similar to the skills of the Brazilian children for all operations. Thus, the conditions of the Brazilian study may have been replicated in our study only in division.

Second, U.S. children may have a good mental strategy available for solving 100 ÷ 4 because of its relationship to the monetary system; the context in which the problem is embedded serves to activate the child's knowledge about the money system, and the child can implement a direct mapping of the problem onto his knowledge of that system. Other division problems, for which the children did not have a written strategy or some way to map the problem onto a well-known quantitative system (e.g., money), often were not even attempted. It is plausible that the Brazilian children were approaching most of the problems in the same way that the U.S. children were approaching the problem 100 ÷ 4. That is, it is possible that the Brazilian children may have had more experience calculating sums of money and, therefore, more recourse to the kinds of mapping strategies seen in the United States for the problem 100 ÷ 4.

Although Carraher et al. did not describe their participants' backgrounds in detail, they said that the children were from a poor area and varied widely in age, a result of starting school late or repeating grades. Because Carraher et al.'s earlier (1985) research among poor children from the same city in Brazil investigated market vendors, it may be that at least some of the children in this study had had market experience. If the children did have market experience in manipulating quantities and in calculation with money, more problems may have been responded to in a way similar to that found for 100 ÷ 4 in the U.S. sample.

The question of the relationship between strategy choice and problem context still remains. The interesting question here may not be why the U.S. children do not use context cues in solving quantitative problems, but rather why the Brazilian children *do* use these cues. Word problems, and even store problems of the type presented in the study, are still largely, or only, encountered at school by U.S. children. The embedding of numbers in a word problem does not really transform the problem into the kind of real-world problem encountered in practical, nonschool problem-solving situations. Word problems are as much artifacts of schooling as are symbolic computation problems and, thus, should perhaps not be expected to activate real-world knowledge. Yet not only did the Brazilian children approach the word problems differently from the computation problems, but the U.S. children did also under certain conditions.

In sum, we are left with a situation apparently more complex than that reported by Carraher et al. On the one hand, simple descriptions of strategies as oral or written, informal or formal, do not adequately capture the approaches taken by our subjects in solving the problems. On the other hand, the relationship between context and strategy choice seems complicated, mediated by factors such as the specific numbers used in the problem or the children's facility with algorithms in general. To clarify these issues, a second study was designed. The goal of the second study was to investigate the conditions under which school-type word problems serve to activate knowledge children possess of cultural systems of quantification and thereby to elicit strategies, such as mapping, making use of this knowledge.

STUDY 2: FACTORS INFLUENCING ACCURACY AND STRATEGY CHOICE IN THE SOLUTION OF WORD PROBLEMS

The results from the replication study suggest that it is not simply problem context, but rather a complex array of factors that influences a child's strategy choice and successful solution of a problem. We begin by discussing two such factors. One factor is the quality of the numbers themselves: Some numbers are just easier to work with than others. The second factor is the degree to which numbers can be mapped onto easily accessible cultural systems such as money.

Certain numbers are easier than others to manipulate within the context of our number system, and different numbers suggest different strategies for manipulation. In our replication study, for example, many children solved the problem 115 + 15 mentally (and correctly), regardless of context. This problem is easy to solve, not because it is easily mapped onto a cultural system of quantification with which children are already familiar (such as money), but because these numbers are easily manipulated within the base-10 system. Other problems are difficult to solve mentally but pose no problems to children with good knowledge of school-taught algorithms. The problem 185 + 68 is one example. Not a single child attempted to solve this problem mentally, yet not a single child solved it incorrectly. Thus, within the confines of our numerical system itself, without any reference to concrete quantities, some problems are easy to solve if school-taught algorithms are invoked (assuming one knows the algorithms), and some are easy to solve by means of other strategies.

Furthermore, particular numbers vary in the degree to which they can be mapped onto particular contents. When numbers are embedded in a problem whose content provides a meaningful context for those numbers in particular, the combination of numbers and context allows children to activate knowledge that helps in solving the problem (e.g., knowledge of the money system). Those same numbers embedded in a problem whose content does not suggest a possible referent may not activate knowledge that could facilitate problem solution. For example, the problem 100 ÷ 4 embedded in a word problem about marbles may be solved using strategies different from those invoked when 100 ÷ 4 is embedded in a word problem involving money, where its relationship to the monetary system is made salient.

The second study was designed to investigate the interactions of particular numbers and problem contents in activating children's real-world knowledge, thus influencing their strategy choices. In particular, we wanted to determine whether children would rely on their knowledge of nonschool cultural systems of quantification when both numbers and problem content related to that system, and whether they would be more successful in solving problems having a match between problem number and problem content compared with problems having no such match.

Our hypothesis was that children would use a mapping strategy for those problems in which knowledge of the system itself could easily be used to solve the problem, that is, problems in which the numbers and the problem content matched. The answers obtained by using mapping strategies should also be almost error-free, and mistakes should be predictable from the nature of the mapped system. In addition, we hypothesized that children would be less successful in solving problems having a mismatch between number and problem content. We also tested whether the ease or difficulty of performing certain manipulations on the numbers themselves influenced strategy choice, regardless of context. Finally, we developed a more detailed categorization of strategies used by U.S. children to better study the relation of strategy use to problem context, to the numbers in the problems, or to both.

Method

Children. Children from two different schools participated in this study: School 1 is a private laboratory school run by a university; School 2 is a parochial school. Both schools are in the same Chicago neighborhood. Fifty-five third graders and 48 fourth graders were tested in School 1, and 32 fourth graders were tested in School 2. Within each school, children were randomly assigned to one of three experimental conditions. Results were pooled across schools and grades for a total of 135 children participating in the study.

Design and materials. The study involved one testing session per child, during which each child solved the same set of 12 problems. The first four problems presented to every child were control problems, two multiplication and two division, that were always presented as symbolic computations. The next eight problems were presented in one of three different contexts, with each child being randomly assigned to one of the three contexts. The first context was *computation*; all problems in this context were presented as symbolic computation problems. In the remaining two contexts, the problems were embedded in standard, school-type word problems. In one context, the eight word problems all involved *time*, whereas in the other context, the problems involved *money*.

The numbers constituting the 12 problems were chosen as follows. The control problems, which were always presented as symbolic computations, consisted of two relatively easy problems and two relatively difficult ones. The eight problems that were varied across context consisted of two problems with numbers that could be mapped easily onto our money system (7×25 and $75 \div 3$), two with numbers that could be mapped easily onto the analog clock (8×15 and $45 \div 3$), two with numbers that were relatively easy but that could not be mapped onto either time or money (5×40 and $40 \div 2$), and two with numbers that were relatively hard but not easily mapped onto time or money (7×19 and $78 \div 6$). The easy and hard problem numbers were chosen from a larger set that had been pretested with a group of fourth graders at a third school; they were asked to solve a set of computation problems and to evaluate the problems in terms of difficulty.[4] Table 5 presents the 12 problems as they appeared in each of the three contexts.

Four different pairings of particular numbers with particular word problems were constructed, and within each operation (i.e., multiplication or division), these pairings were counterbalanced across subjects. To summarize the procedure, four control problems were presented to all children. Then

[4]Throughout the remainder of this article, we capitalize the problem types. This is done to reduce confusion between, for example, Money problems (the numbers 7×25 and $75 \div 3$) and the money context (the content of the word problems themselves).

TABLE 5
Problems and Contexts of Study

Problem Numbers					
Control					
Easy	*Hard*	*Money*	*Time*	*Easy*	*Hard*
7 x 20	11 x 13	7 x 25	8 x 15	5 x 40	7 x 19
80 ÷ 2	70 ÷ 5	75 ÷ 3	45 ÷ 3	40 ÷ 2	78 ÷ 6

Contexts

Time

1. Mary watched ___ TV shows in a row. Each show lasted ___ minutes. How long was Mary watching TV?
2. Kim read ___ books. Each book took ___ minutes to read. How long did it take to read all the books?
3. The baseball game lasted ___ minutes. There were ___ innings in the game. Each inning lasted the same amount of time. How long was each inning?
4. It took Raphael ___ minutes to walk to school. He walked ___ blocks. How long did it take to walk each block?
5. Sue walks ___ blocks to school every day. It takes Sue ___ minutes to walk one block. How long does it take to get to school?
6. John ate ___ Big Macs. It takes John ___ minutes to eat a Big Mac. How long did it take him to eat them all?
7. Shirelle read for ___ minutes and read ___ books. How long did it take Shirelle to read each book?
8. Vicki baked for ___ minutes and baked ___ batches of cookies. How long did it take Vicki to bake each batch?

Money

1. Mary brought ___ cans of pop. Each can cost ___ cents. How much did Mary pay?
2. Kim rode the bus ___ times. Each ride cost ___ cents. How much did Kim spend on the bus?
3. Raphael gave ___ cents to ___ children who weeded his garden. How much did each child get?
4. It costs ___ cents to buy ___ packs of gum. How much does one pack of gum cost?
5. Sue bought ___ packs of baseball cards. Each pack cost ___ cents. How much did Sue pay?
6. A group of ___ children went to the museum together. It cost each child ___ cents to get in. How much did the group of children have to pay?
7. It costs ___ cents for ___ children to go on a roller coaster at the fair. How much does it cost for each child?
8. Vicki paid ___ cents for ___ apples. How much does one apple cost?

children were presented with four types of additional problems that differed according to the numbers used—Time, Money, Easy, and Hard—in one of three problem contexts—computation, time, or money. Thus, a child in the time context was given eight problems in the context of time: two problems in which the numbers matched (i.e., were expected to be easy in) the problem context, time; two problems in which the numbers mismatched the time context but would have matched a money context and, therefore, were expected to be difficult in a time context; two problems in which the numbers did not map onto any culturally transmitted symbol system but were easy to calculate because of relations between numbers in a base-10 system; and two problems in which the numbers were difficult to calculate for the same reason. A child in the money context was confronted by the same setup, only the matching and mismatching problems were reversed. A child in the computation context was presented with problems not embedded in a word context; problems using both Time and Money numbers were expected to be equally challenging.

Procedure. Problems were presented orally to each child in a one-on-one session with the experimenter. The experimenter repeated the problem as many times as the child requested. Children were told that they did not have to show their work, and that if it was easiest to solve a problem in their heads, they should do so. Children were instructed to say the answer aloud after solving the problem. At that point, the experimenter made sure she understood the solution method before the next problem was read. If solution strategy was unclear from the scratch paper, or if the child had solved the problem mentally, the experimenter asked the child how the problem was solved, using neutral probes until the solution steps were clear. The experimenter recorded the answer and solution protocol, which was either the child's verbatim explanation or a record of the written procedure.

Coding. As in Study 1, each problem solution was coded for correctness and for the strategy used to get the answer. The coding of strategies differed in significant ways from that employed in the previous study, however. Because this second study was not a replication, we were able to derive inductively a new system for coding strategies. Many children tried two or three different strategies before arriving at a final answer, but only the final strategy attempted was coded. If a mixture of strategies was used to arrive at the final answer, the strategy that contributed most to the outcome was specified and coded. Ten strategies were identified, with an 11th category for other, uncodable solutions (typically because the experimenter did not record enough information at the time of testing). The 10 strategies are as follows:

1. *Decomposition (DECOMP).* The child broke down the problem into a series of easier problems to which the answers were already known. Examples of this include solving the problem 7×20 by solving $(7 \times 10) + (7 \times 10)$, or solving 5×40 by solving 5×4 and adding a zero.
2. *Repeated addition (READD).* The child used an iterative counting

strategy to solve the problem. This category also includes the use of tallies on paper. The strategy was used for both multiplication and division. Using repeated addition to solve a multiplication problem, 15 × 8, might involve adding 15 eight times, making 8 sets of 15 tally marks, or adding 15 + 15, then 30 + 30, then 60 + 60. Solving a division problem such as 70 ÷ 5 using repeated addition might consist of counting by 5s to 70 and then counting the number of 5s used.

3. *Algorithm (ALG).* The child used a school-taught algorithm involving the manipulation of symbols. An example of problem solving using this strategy for 8 × 15 is "8 times 5 is 40, put down the zero and carry the 4; 8 times 1 is 8 plus 4 is 12. Put down the 12; the answer is 120."

4. *Mapping (MAP).* The child used his or her knowledge of a cultural system such as money or time to solve the problem by mapping the numbers in the problem onto that system. For example, a child solved 75 ÷ 3 by thinking in terms of three quarters as equivalent to 75 cents. Only one child mapped onto something other than money or time; he thought of 7 × 25 in terms of touchdowns in a football game.

5. *Retrieval (RET).* This was coded if, in response to the experimenter's probe of "How did you get that?", the child was unable to describe any protocol other than "I just know the answer." This strategy was used most often on the easy problems of 80 ÷ 2 and 40 ÷ 2.

6. *Estimation (EST).* This strategy was coded if the child made an explicit reference to thinking of a number that might be close to the answer, but made no real attempt to manipulate the numbers in any way. An example of this is an answer of "about 21 or 22" for the problem 45 ÷ 3, and an explanation that the child estimated the answer. At times, however, children made sophisticated efforts at estimation, arriving at the correct answer by trying several numbers until one fit. This was also coded as estimation.

7. *Guess (GUESS).* The child who guessed made no effort to think about the numbers, but simply answered with what seemed to be the first number that came into his or her head. This strategy was coded when the child could give no explanation for solution strategy other than "I just guessed."

8. *Wrong operation (OPERAT).* The child used the wrong operation in solving the problem. Although not technically a strategy, errors in operation reflect a child's faulty representation of the problems. This was considered important because certain contexts and problem numbers may aid or hinder correct representation of the problems. The operation used and the strategy used on the operation were not coded.

9. *Opaque (OPAQ).* The child used some kind of seemingly idiosyncratic strategy that had its own rules and processes but did not fit into any of the categories already listed. The child using an opaque strategy does *something* to arrive at an answer but does not use a conventional strategy. For example, OPAQ was coded when a child gave an answer of 108 for the problem 78 ÷ 6 and reported the following protocol, "I divided it up into little pieces; I took

all the 5s out of the 8s, and then the 10s; and I had to count all of the 3s in there."

10. *Skipped problems (SKIP).* The child abandoned the problem without giving an answer. (Skipped problems were coded as incorrect and were included in summary computations of proportion correct.)

One strategy was coded per problem. Intercoder reliability for coding strategies, as measured by proportion of a randomly chosen subset of problems that was coded identically by two independent coders, was .97.

Analyses. Performance on the four control problems was analyzed first to determine if the three groups were comparable. A one-way ANOVA revealed no significant effect of group, $F(2, 132) = 0.066$, $p = .94$. Thus, differences among contexts in subsequent analyses may be assumed to be due to experimental condition rather than to preexisting differences among the groups.

Mean proportion correct as a function of condition and problem numbers was analyzed in a series of repeated-measures ANOVAs. Two separate analyses were conducted, each based on half the eight experimental problems. The first treated numbers used Time versus Money as a repeated measure, and context as a between-subjects variable with three levels. The second analysis was identical to the first, except that Easy versus Hard problems were compared instead of Time versus Money.

Individual summary variables for each of the solution strategies were constructed as the proportion of problems of each type on which the child employed a given strategy. Therefore, for the two problems using Money numbers, children were given a score of 0, .5, or 1 for each strategy, depending on whether they never used that strategy, used it once, or used it to solve both problems. The same was done for the two problems using Time numbers, the two Easy problems, and the two Hard problems. The mean proportions of use of each strategy were then analyzed in a series of repeated-measures ANOVAs similar to those carried out for proportion correct, comparing problems using Time versus Money numbers and Easy versus Hard numbers across contexts.

Results

Mean Proportion Correct

To ascertain whether problems in which the numbers matched the context were easier to solve than problems in which there was a mismatch between the numbers and the problem context, we compared the proportion of problems using Money and Time numbers solved correctly across the three contexts. It was expected that a greater proportion of problems using Money numbers than Time numbers would be solved correctly in the money con-

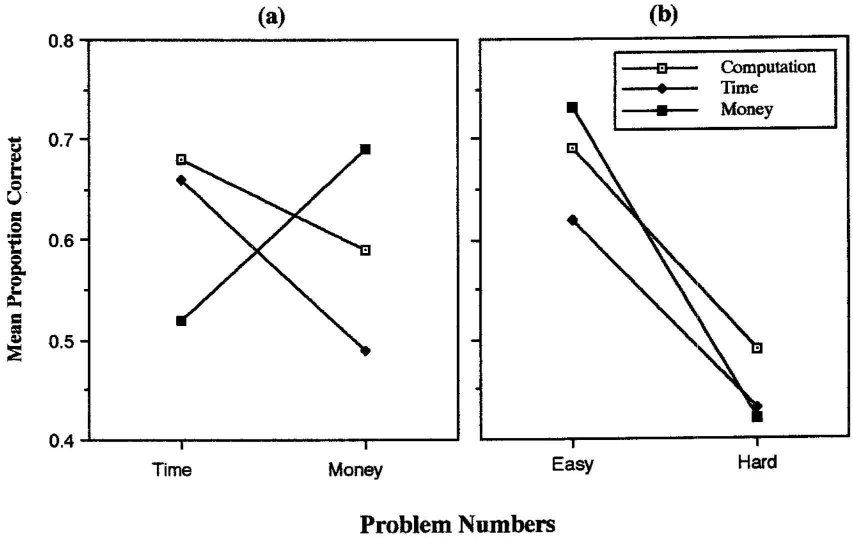

FIGURE 4 Mean proportion of problems solved correctly, broken down by context and problem numbers. Panel (a) presents Time versus Money problem numbers; panel (b) presents Easy versus Hard problem numbers.

text, and that a greater proportion of problems using Time numbers than Money numbers would be solved correctly in the time context.

The results are presented in the left panel of Figure 4, in which we present mean proportion correct by problem numbers and context. The results of the ANOVA revealed that, although there were no significant main effects of context or problem numbers, there was a significant interaction between problem numbers and context, $F(2, 132) = 9.05$, $p < .001$. As is evident in Figure 4 and confirmed by post hoc tests of simple effects, children in the time context solved more problems with Time numbers than Money numbers correctly, $F(1, 132) = 5.70$, $p < .05$, whereas children in the money context solved more problems with Money numbers than Time numbers correctly, $F(1, 132) = 5.12$, $p < .05$. Children who solved the problems in the computation context showed no significant difference in proportion correct for problems using Time numbers versus Money numbers. Thus the context aided the children in solving the problems, but only when context was matched by the problem numbers.

Problems using Easy and Hard numbers were analyzed in the same manner; results are shown in the right panel of Figure 4. Again, there was no overall context effect, and there was no Problem Type × Context interaction. There was, however, a significant effect of problem numbers, $F(1, 132) = 113.71$, $p < .001$. Predictably, a much greater proportion of problems using Easy than Hard numbers was solved correctly.

Strategy Use

Control problems. An analysis of strategy use on the control problems was carried out to insure that children in all three conditions exhibited the same pattern of strategy use for identical problems. Repeated-measures ANOVAs were done for the mean proportion of use of each strategy with Easy versus Hard control problems as a repeated measure and context as a between-subjects factor. Effects of problem numbers were found but no context effects and no interactions; the pattern of strategy use for the control problems was similar across the three contexts.

After insuring equivalent patterns of strategy use across contexts for the control problems, strategy use on the remaining problems was analyzed separately for problems that used Money versus Time numbers and Easy versus Hard numbers, with problem numbers as a repeated measure and context as a between-subjects variable. This was done because this study was designed to investigate the potential differences in strategy choice, depending on the numbers in the problem (Money, Time, Easy, or Hard); therefore, the analyses are presented separately for problems using Money versus Time numbers and Easy versus Hard numbers. Mean proportions of use of each strategy for the problems using Money versus Time and Easy versus Hard numbers are reported in Table 6.

Problems using Money numbers versus Time numbers. Originally, we had hypothesized that children would use a mapping strategy when they received problems with Money numbers in a money context and, likewise, would use mapping when they received problems with Time numbers in a time context. This was partially true. When analyzing for the mapping strategy, we found significant main effects for context, $F(2, 132) = 5.50$, $p < .01$, and for problem numbers, $F(1, 132) = 23.72$, $p < .01$. Children used more mapping strategies for problems using Money numbers than for problems using Time numbers, and they used more mapping strategies in the money context than in either the time or computation contexts. Furthermore, a significant Problem Number × Context interaction was found for the use of mapping strategies, $F(2, 132) = 7.64$, $p < .01$, indicating that the use of the mapping strategy depended on both the numbers in the problem and the context in which the problem was presented. Specifically, children used the mapping strategy most frequently if they were given a problem with Money numbers in a money context. No parallel pattern was found for problems with Time numbers in a time context. Thus, as hypothesized, the provision of a money context facilitated the activation of the relevant knowledge only when the numbers could be mapped onto the money system, and counter to our hypothesis, the provision of a time context did not facilitate a comparable activation, even when the numbers could easily be mapped onto an analog clock.

TABLE 6
Mean Proportion of Problems Solved Using Various Strategies According to Context and Problem Number

	Strategy[b]										
Context[a]	*DECOMP*	*READD*	*ALG*	*MAP*	*RET*	*EST*	*GUESS*	*OPERAT*	*OPAQ*	*SKIP*	*OTHER*
Money numbers											
Computation	.11	.11	.48	.09	.02	.03	.01	.05	.03	.04	.01
Time	.05	.13	.32	.07	.04	.03	.01	.20	.03	.03	.09
Money	.04	.08	.26	.28	.07	.03	0	.10	0	.06	.09
Time numbers											
Computation	.10	.17	.50	.01	.01	.07	.01	.02	.06	.05	0
Time	.08	.17	.35	.03	.01	.10	0	.14	.03	.03	.05
Money	.06	.17	.39	.01	.04	.08	0	.16	.02	.06	.02
Easy numbers											
Computation	.15	.15	.34	0	.17	.01	0	.10	.02	.01	.05
Time	.07	.20	.21	0	.22	.02	.01	.18	.02	.01	.07
Money	.10	.16	.22	0	.32	.01	0	.08	.02	.01	.08
Hard numbers											
Computation	.07	.14	.48	0	.02	.07	.02	.03	.05	.09	.03
Time	.03	.16	.41	0	0	.04	.01	.12	.04	.14	.03
Money	.08	.10	.40	0	0	.08	.02	.09	.02	.13	.08

[a]Computation, $n = 44$; time, $n = 46$; money, $n = 45$.
[b]See Coding section for explanation of strategies.

We found other strategy differences as well. Three strategies were used more often on problems with Time numbers than Money numbers: repeated addition, $F(1, 132) = 15.18$, $p < .01$, estimation, $F(1, 132) = 8.32$, $p < .01$, and algorithms, $F(1, 132) = 6.79$, $p < .01$. The problem 8 × 15 seemed particularly conducive to a repeated addition strategy, and 45 ÷ 3 was the problem most often solved through estimation. Apparently the numbers here, regardless of their relation to the analog clock, elicited these strategies.

Although use of an algorithm was more frequent for problems using Time numbers than Money numbers, algorithms also were used differentially, depending on the problem context—that is, a Problem Context × Problem Type interaction just missed significance, $F(2, 132) = 2.80$, $.05 < p < .10$. Specifically, children were more likely to use a school-taught algorithm to solve a problem when that problem was presented in a computation context, especially when the numbers themselves did *not* represent quantities associated with the money system.

We also found a significant main effect for problem context for using the wrong operation, $F(2, 132) = 5.86$, $p < .01$. Children were more likely to use the wrong operation if the problem was presented as a word problem (i.e, in a money or a time context) than if it was presented in a computation context. Thus, it appears that children are more likely to pay attention to the correct operation if the sign for that operation is presented in the problems, as is the case for problems presented in the computation context. Moreover, we found a significant interaction between problem number and context for using the wrong operation, $F(2, 132) = 3.86$, $p < .05$. A larger proportion of errors of operation was made on the problems with Money numbers in the time context and on the problems with Time numbers in the money context. Because an error in operation may indicate faulty understanding of a word problem, it appears that the mismatch between problem content and the numbers in the problem gave rise to more faulty interpretations of a problem, whereas a match between problem content and numbers had the opposite effect.

Even a quick perusal of Table 6 reveals that some strategies were used infrequently for problems using both Money and Time numbers. It appears that children were reluctant or unable to retrieve an answer, guess a solution, use an opaque strategy, or skip a problem. Thus, it is not surprising that no significant differences were found for use of these strategies for problems with Money or Time numbers.

Problems using Easy versus Hard numbers. Unlike the hypothesis generated for problems with Money and Time numbers, we did not expect children to use mapping strategies for problems with Easy or Hard numbers. Indeed, not one child used a mapping strategy for either Easy or Hard problems.

We did, however, find some strategy differences for solving problems using Easy versus Hard numbers. The most striking difference between these problems was that children quite often used retrieval for problems with Easy numbers but never for problems with Hard numbers, $F(1, 132) = 107.56$, $p < .001$. Furthermore, children were most likely to retrieve a solution if the problem was in a money context and least likely to retrieve if the problem was in a computation context.

We found other strategy differences as well. Three strategies were used more often on problems with Hard numbers than Easy numbers. Children were more likely to skip a problem, estimate, or use an algorithm when faced with Hard numbers than when faced with Easy numbers: $F(1, 132) = 19.39$, $p < .001$, for skipped problems, $F(1, 132) = 9.52$, $p < .01$, for estimated answers, and $F(1, 132) = 24.34$, $p < .001$, for using algorithms.

We also found a significant main effect for problem context for using the wrong operation. As for the problems using Money and Time numbers, children were more apt to use the wrong operation for Easy and Hard numbers if the problem was in a money or time context than if it was in a computation context, $F(2, 132) = 3.01$, $p < .05$. Again this demonstrates that children are more likely to carry out the correct operation if the sign for that operation is present in the problem itself, as is true for problems presented in the computation context.

The relationship of strategy use and solution accuracy. We have shown that children correctly solved more problems with Money numbers than Time numbers in the money context. We have also shown that children in the money context solved problems with Money numbers using a mapping strategy more often than they solved problems with Time numbers using this strategy. But we have not shown that the particular problems with Money numbers in which mapping strategies are used are solved more successfully than problems in which such strategies are not used. To ascertain the relationship between accuracy and strategy use at the level of the individual problem, the proportion of problems with Money numbers solved correctly using a mapping strategy versus the proportion of problems solved correctly using all other strategies was calculated separately for each context. Mapping was found to be a highly successful strategy, regardless of context. The proportions of problems with Money numbers solved correctly when a mapping strategy was used were .92, .83, and 1.0 for the computation, time, and money contexts respectively. The proportions of these problems solved correctly when all other strategies were used were .57, .46, and .58 for the three contexts. Therefore, it appears that, for problems with Money numbers, strategy used rather than context was the primary determinant of solution accuracy. The effect of context most likely works by causing different strategies to be accessed and used. However, this analysis does not work so neatly with problems in which Time numbers were used. Although children in the time context were more successful in solving problems with Time numbers than Money numbers, this difference was not clearly tied to differential strategy use. This leaves open the possibility that other mechanisms—such as facilitation of problem representation—may also mediate the effect of context on problem solving.

Discussion

The purpose of this study was to examine the effects of particular numbers and the contexts in which numbers are presented on children's mathematical problem-solving performance. A subset of problems provided children with the opportunity to map numbers onto specific contexts, which could facilitate solving the problems. We found that children took advantage of this opportunity when they were provided with problems using Money numbers in a money (word problem) context, but not when they were provided with problems using Time numbers in a time context. Furthermore, children did not map problems using Money numbers onto money when the problems were presented in the time or computation contexts. Thus, it was possible to elicit children's knowledge of a nonschool quantitative system to help them solve mathematical problems, but only when the particular numbers were appropriate for the context.

The results for problems using Time numbers in time contexts were different from the results for problems using Money numbers in money contexts. Although children did better solving problems with Time numbers in the time context than problems with Money numbers in the time context, the patterns of strategy use do not suggest a clear reason for this. Several explanations of this difference seem plausible. First, it is possible that children in this sample were relatively unfamiliar with analog clocks due to the increasing popularity of digital clocks. If this is the case, calculating with 15-min chunks would not be a salient property to use for mapping. Second, the actual numbers chosen for the Time problems were based on quarter hours (15 min) rather than half hours (30 min) to avoid confounding problems with Time numbers and Easy numbers. It is possible however, that children who would be likely to map some problems with Time numbers onto an analog clock actually manipulate time in terms of half hours more frequently than quarter hours. Given the nature of this study, we cannot determine whether either of these explanations is accurate. In any case, there seems to be some advantage in the match between problem number and context that allows for more successful solution of a problem.

The results indicated that problems using Easy numbers were both easier to solve and elicited different problem-solving strategies than problems using Hard numbers. In general, these findings were not surprising. However, some of the particular findings deserve notice. As stated earlier, all the strategies were empirically derived (i.e, the coding system was based on children's actual responses). With this in mind, it is interesting to note that certain strategies used by children to solve problems using Money or Time numbers were never used for problems with Easy or Hard numbers and that some strategies, such as the application of school-taught algorithms, were used more on problems with Hard than Easy numbers. Thus, it appears that the numbers themselves, apart from problem context, can compel children to select certain problem-solving strategies.

We also found that the context—a money word problem, a time word problem, or symbolic computation—can affect the strategy that a child will choose to solve a problem. However, there were few pure context effects that were not accompanied by an interaction with problem numbers. Additionally, children were more easily led astray by word problems; in other words, they more often chose the wrong operation if the problem was embedded in words than if the problem was presented in numerical symbols. At the same time, some aspects of word problems obviously facilitated children's problem solving. We found that retrieval was facilitated by placing the problem in the context of a word problem (especially problems with Easy numbers in the money context) and, as mentioned earlier, mapping was encouraged when problems using Money numbers were placed in the context of a money word problem.

CONCLUSION

At the beginning of this study we asked why it is that word problems—problems that are constructed, in part, to provide children with out-of-school contexts for solving problems—do not generally activate real-world knowledge representations and procedures that could be used in the solution of the problems. Study 1 showed that children will solve problems differently depending on context, but the relevant context is more specific than "real world" versus "computation." Study 2, picking up on what we suspected were important contextual cues (i.e., numbers that could be readily mapped onto a culturally supported system, such as money, and real-world knowledge evoked by the context itself), validated that children make use of some cues but not others. Indeed, in Study 2, it became obvious that the relationship between strategy choice and problem context is a more complex one than was posited in Study 1. The numbers interact with problem content in a way that may or may not facilitate successful solution of the problem. It is apparent that the content of word problems can induce children to use knowledge of a culturally constituted system of quantification, such as money, to help them in solving problems, but only if the particular numbers used in the problem make it possible to do so. Likewise, particular numbers that lend themselves to being mapped onto a system such as money will induce such mapping, but generally only if the cultural system is suggested by the problem content. Mapping problems onto a quantitative system such as money does increase the likelihood of a problem's being solved successfully.

Clearly, we must move beyond the simple equation of school with formal, algorithmic, written, and context-free; and of outside school with informal, intuitive, oral, and context-full. Things are not so simple. In its most successful moments, school enables children to access the wealth of real-world knowledge that they have accumulated outside school. And as for the real world itself, there are surely times when the strategy of choice might be one that was learned in school, if only it had been learned in such a way that it could be linked to the problem at hand, and then performed efficiently and accurately. One possibility suggested by our study to explain why children do not use school-taught algorithms in real-world problem-solving contexts is that they have not mastered the algorithms to a sufficient extent. When they encounter a problem with hard numbers, they often use the school-taught algorithm. School-taught algorithms are most useful for problems with odd and difficult numbers, because with such numbers, other strategies rarely come to mind. Yet, ironically, it is just such numbers that diminish children's confidence in being able to correctly apply the algorithm, and indeed, they tend to make errors in applying algorithms to problems with difficult numbers.

In reading the studies showing how school-taught strategies are not used outside school, one often wonders whether the implication is that schools might just as well not teach algorithms at all. This is definitely not implied by the results we have reported. We have shown that although in some cases the school-taught algorithm is simply not the strategy of choice, in other cases it would be a welcome strategy if only it were truly available. Providing contextual cues may help children to access alternative strategies for problem solving, and this seems to be a good tactic. But it also seems that algorithms should be taught, though much better than they are now. If students have really mastered algorithms, they will be able to use them in cases in which they would be most useful. Sometimes, contextual strategies are used not because they are the best ones to use, but because students have no other strategies available. This does not mean that this limitation is an inherent characteristic of the human problem-solver, but rather that it is a failure of our schools to educate well. Linkages between the worlds in and out of school are important; each world has something to offer the other. The intriguing problem that we have only begun to address in this study is that of exactly how children can be encouraged to invoke their knowledge of nonschool contexts to understand and solve mathematical problems of all types.

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