# Teachers' Responses to Student Mistakes in Chinese and U.S. Mathematics Classrooms

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**Abstract**

The treatment of errors in mathematics classrooms has gained attention in recent years, with many researchers suggesting that errors should be used as starting points for student inquiry into mathematics. In the study reported in this article, we examined how teachers used discourse around errors to generate inquiry by looking at the treatment of mistakes in U.S. and Chinese elementary mathematics lessons. To do so, we videotaped 44 lessons from Chinese and U.S. first-grade ($n = 15$), and fourth- and fifth-grade ($n = 29$) classrooms and also interviewed the teachers of the lessons. We separated the lessons by the topic taught (place value or fractions) and analyzed them for frequency of students' errors and the types of teachers' responses to these errors. Results indicated that U.S. and Chinese students made errors at similar frequencies. However, the teachers in the 2 countries responded to errors differently. In particular, the U.S. teachers made more statements about errors than the Chinese teachers, who instead asked more follow-up questions about errors. Relying on qualitative analysis of teacher interview and in-class statements about errors, we shed light on both how teachers used errors for inquiry and what teachers believed about errors.

Research on student errors in mathematics has had a complicated and often controversial history. In 1922, Thorndike proposed his associationist theories of mathematics and fostered the belief that errors are the result of a lack of drill and practice of "number facts" such as the addition and multiplication of single-digit combinations (Resnick & Ford, 1981). In fact, educators saw the presence of errors as a menace that could cause students to form false associations between numbers.

In the 1970s, errors were still seen as problems that should be eradicated, but researchers such as Lankford (1972) developed an interest in the analysis of errors as a method for understanding the causes of common procedural mistakes. By examining the procedures that successful and unsuccessful students used in computation, Lankford found that the errors of unsuccessful students were systematic. This finding gave rise to research on the "bugs" students make in computation (Brown & VanLehn, 1980), with the implication that teachers could diagnose bugs in student work and help students move beyond these computational errors (Resnick & Ford, 1981).

Only recently have researchers started to view errors as resources for promoting learning rather than simply as diagnostic tools. Since the National Council of Teachers of Mathematics (NCTM) published their *Professional Standards for Teaching Mathematics* in 1991 and emphasized the importance of student discourse in mathematics learning, researchers of mathematics teaching have emphasized the role that student mistakes or incorrect conjectures can play in promoting debate about mathematical topics and in assessing student understanding of mathematical topics. In particular, Borasi (1994) suggested that mistakes can be "springboards for inquiry" into mathematical concepts. In examining errors in this context, Borasi found that student mistakes can propel students' engagement in mathematical discussions, providing a context to examine and critique their own and others' thinking.

Writing about the importance of discourse in mathematics education, Ball (1991) emphasized that errors can be a window into student understanding. She argued that teachers must move beyond "right" and "wrong" answers and look instead at student thinking. In doing so, teachers should ask students how and why they get right answers and should try to deduce the conceptual thinking behind wrong answers. To facilitate this, teachers need to make students feel that it is safe to talk in the classroom, particularly to give wrong answers and to change their minds. Continuing this train of thought, Ball (1993) suggested that one of the purposes of extended discussion in mathematics classrooms is to uncover the misconceptions about mathematics that students bring with them to the classroom.

Work in cognitively guided instruction (CGI) has provided additional conceptual support for Ball's emphasis on using errors as a gauge of student understanding. In particular, CGI focuses on mathematics teachers monitoring student thinking and relying on knowledge of student thinking to guide instruction (Fennema et al., 1996; Fennema, Franke, Carpenter, & Carey, 1993).

Other researchers have built on the idea that errors can promote productive mathematical debates. Lampert (1992) and Rittenhouse (1998) have written about creating risk-free spaces in mathematics classrooms where students feel comfortable providing conjectures rather than fully formed thoughts. Lampert suggested that student debates about mistakes mimic the discourses of "authentic" mathematicians who debate in similar ways. Expanding on the idea that student freedom to produce errors is crucial for successful discourse in mathematics classrooms, Blunk (1998) and Webb and Mastergeorge (2003) have discussed the role that small groups can play as environments for students to express their misunderstandings and to help other students work through their mistakes. In particular, Webb and Mastergeorge posited that students could learn from their errors by asking their peers for specific explanations, receiving high-level help from their peers, and practicing what they learned.

Cobb, Wood, Yackel, and McNeal (1992) have also delineated how classroom discussion of errors became part of the sociomathematical norms of two elementary school mathematics classrooms. According to Cobb et al., the sociomathematical norms of one classroom emphasized mathematical activity simply as following procedural instructions. In this classroom, the teacher and students saw errors as transgressions of mathematical norms. The teacher and students in the class responded to errors by quickly correcting each error or the procedure that led to it. In contrast, Cobb et al. also described a classroom that emphasized mathematical truths. In this classroom, students who made errors understood that they should justify or explain the reasoning behind the error. In addition, other students accepted the presence of errors and knew that they should challenge all answers and conjectures, whether they were right or wrong.

Kazemi (1998) provided evidence that discussion of errors may lead to greater achievement among students in mathematics. She analyzed teachers who practiced "high-press" and "low-press" discourse in their mathematics instruction. High-press teachers invited discussion about student errors and created an atmosphere of mutual respect between students where it was safe to err; in contrast, low-press teachers limited discussion of errors. Kazemi directly linked high-press teaching to student achievement. Although the components of high-press teaching include more than just the treatment of errors, Kazemi's work suggests that the use of errors as "springboards for inquiry" is a natural part of teaching that emphasizes discourse in mathematics.

Clearly, the wealth of research about the importance of rich conversations surrounding errors has implications for teaching mathematics. The question becomes: What exactly are these implications? Simply knowing the utility of a tool for mathematics instruction is not the same as implementing it effectively. For instance, using interesting and complex problems in mathematics may be a good strategy, but assigning such problems to students will not automatically improve their understanding. Likewise, using manipulatives has long been a staple of mathematics instruction, but placing blocks or ten-rods on students' desks will not guarantee conceptual clarity. In this same vein, the question of the study reported in this article became: What is known about how errors are treated and used in classrooms and what insights does this knowledge offer into how teachers can use errors to promote further inquiry in classroom discourse?

## A Springboard for Inquiry into Errors: Cross-Cultural Teaching Practices

To investigate the use of errors for inquiry in classroom discussion, we decided to examine how teachers already implemented such discussions. We wanted to look closely at how errors were handled in lessons in a U.S. sample, and, to clarify and strengthen our perceptions, we expanded our view to include lessons Chinese teachers presented. As the ethnographer Wolcott (1999) suggested, looking at other cultures allows members of a culture to see their own particularities that might otherwise go unexamined. In this study, we adopted such an approach.

Prior research has set a precedent for using a cross-cultural lens for looking at the discourse surrounding errors. As early as 1965, Nuthall and Lawrence devised a coding scheme to describe the types of errors students in New Zealand and the United States made. More recently, Santagata (2004) examined the affective responses U.S. and Italian teachers made to public student errors during mathematics lessons. She coded whether a teacher gave a neutral response to an error, "mitigated" the error by telling the student he or she was close to the right answer or excusing the student from responsibility, or "aggravated" the error by criticizing the student for making it. Whereas Italian teachers more often aggravated errors, U.S. teachers more frequently mitigated them. Although neither of these studies relates to the questions we investigated in the study reported in this article—in particular, how teachers use errors to enhance student discussion of mathematics—they do lay important groundwork for using a cross-cultural approach when looking at errors.

The inspiration for the current study stemmed from work done on the treatment of errors in China and the United States. We selected China as a partner for study not only because of its history of high student achievement in mathematics (Fan & Zhu, 2004; Stevenson, Chen, & Lee, 1993; Stigler & Perry, 1988) but also because research has shown that Chinese teachers implement discussions of errors in exemplary ways. Specifically, Stevenson and Stigler (1992) and Wang and Murphy (2004) argued that Chinese teachers use errors to prompt student discussion of mathematical concepts and promote a classroom environment in which students do not feel ashamed to make mistakes.

As an example, Stevenson and Stigler (1992) observed Chinese and U.S. instruction and posited that Chinese teachers used errors more "constructively" than U.S. teachers. For instance, when confronted with a student mistake, Chinese teachers asked other students if they agreed with the mistake or asked questions that directed attention to why the mistake was wrong. In contrast, U.S. teachers often posed the original question to another student. Stevenson and Stigler (1992) related these cultural teaching practices regarding errors to cultural beliefs, stating, "For Americans, errors tend to be interpreted as an indication of failure in learning the lesson. For Chinese and Japanese, they are an index of what still needs to be learned" (p. 192).

Citing Stevenson and Stigler, Wang and Murphy (2004) also analyzed the public discussion of errors in Chinese classrooms. They argued that Chinese teachers respond to errors by "dwelling on them," correcting the error and then asking the students to explain the reasoning behind the error. Linking these practices to culture, they wrote that Chinese students, as opposed to U.S. students, do not experience embarrassment regarding errors. They attributed this lack of embarrassment to cultural beliefs in the authority of the teacher and in the possibilities of learning from mistakes. Wang and Murphy summed up these cultural beliefs in the Chinese adage: "Failure is the mother of success" (p. 120).

Although both Wang and Murphy (2004) and Stevenson and Stigler (1992) provided powerful explanations of the general viewpoints of U.S. and Chinese teachers regarding errors, their observations did not include detailed information about how teachers implemented discussions about errors. To gain a rich understanding of discourse and inquiry surrounding errors, a systematic look at the treatment of errors in the United States and China is required. By looking at teachers in a country who are perceived to handle errors in an exemplary way (China) and at teachers in the United States, we can isolate potentially effective practices in both countries and examine how teachers in different contexts use errors to promote inquiry.

In our investigation, we attempted to answer three questions: Do the instructional practices of Chinese teachers look different from those of U.S. teachers in terms of responses to errors? If so, how are they different? And, finally, what can we learn from these differences? As we try to learn from these differences, we make inferences about which practices may be more and less effective, especially in terms of how teachers use errors to promote further inquiry (Borasi, 1994; Kazemi, 1998; NCTM, 1991). To address these issues, we began by looking at the frequency of errors in classrooms in both countries, by analyzing teachers' responses to these errors, and by closely examining how teachers talk about errors and how those spoken beliefs contribute to in-class practice regarding errors.

Based on Stevenson and Stigler's (1992) and Wang and Murphy's (2004) work, we hypothesized that Chinese teachers' responses to errors would emphasize having students reason about and explain errors, whereas U.S. teachers' responses would focus on correcting errors immediately. However, the major aim of our research was not to draw comparisons between the two countries but to identify practices and beliefs in each country that illustrate the use of student errors as bases for inquiry in mathematics. As we noted in our earlier example of blocks and tens-rods on students' desks, the implementation of an instructional strategy is key in determining whether students benefit from a particular practice. Classrooms are complex places, and we acknowledge that mathematical discourse around errors is a complex event but one that warrants unraveling if educators are to understand what constitutes sound practice in this area of classroom discourse.

## Method

### Videotapes of Lessons

To gather data for this project, we videotaped 46 mathematics lessons in elementary schools in China and the United States. The lessons covered two mathematical topics that are fundamental to early mathematics learning: (1) place value and (2) the manipulation of fractions with different denominators (including adding fractions with different denominators and creating and understanding equivalent fractions with different denominators). We thought it was important to control for the topics discussed so that the frequency and range of possible errors, and therefore possible responses to errors, were limited by the content.

Both the Chinese and U.S. teachers introduced place value in the first grade. However, Chinese teachers introduced fractions with different denominators exclusively in the fifth grade, whereas some U.S. teachers introduced this topic in fourth and others in fifth grade. We videotaped 10 first-grade and 16 fifth-grade Chinese lessons conducted in schools in Beijing, China, and six U.S. first-grade lessons and 14 fourth- and fifth-grade lessons from schools in and around a midsize Midwestern city.

### Teachers

The participating teachers included 10 first-grade teachers (all female) and 14 fifth-grade teachers (12 female, 2 male) from China, and five first-grade teachers (all female) and 12 fourth- and fifth-grade teachers (1 male, 11 female) from the United States. The teachers were a convenience sample in the sense that we videotaped only teachers who agreed to participate. However, these teachers taught in many types of schools (e.g., urban, rural, high prestige, medium prestige), and thus we believe they are representative of the areas from which we selected them in each country. Because we did not randomly select the teachers, we naturally exercise caution in drawing inferences. We limit our analyses to explicating interesting teaching practices rather than to drawing cross-national comparisons.

### Transcribing Lessons

After videotaping the lessons, we transcribed them. We noted sections in the transcripts where the teacher and students were not engaged in whole-class instruction (such as seat work and small-group time) and omitted these to focus our analysis exclusively on errors and responses to errors made during whole-class discussions. We did this for two reasons: (1) it was often hard to hear conversations in small groups or during seat work, so we could not be certain we were consistently hearing and coding errors that occurred during these times, and (2) we were interested in how the teacher's implementation of discourse surrounding errors fostered certain sociomathematical norms (Cobb et al., 1992) regarding errors in each classroom. If the entire class cannot hear a discussion regarding an error, then this type of interaction may influence sociomathematical norms for some students but does not necessarily reflect on how the teacher constructs these norms.

When analyzing the lessons for whole-class time, we found that two U.S. lessons had no whole-class time. In both of these lessons, the teacher worked with small groups on mathematics while the rest of the students worked individually on different projects. This is a form of small-group activity in which the teacher rotates groups to work intensively with four or five students on mathematics. One of these lessons was a place-value lesson, and the other was a fractions lesson. We omitted these lessons from any comparative analyses across the countries; therefore, 44 total lessons were included for analyses.

### Interviews

In addition to videotaping lessons, we asked each teacher to participate in a brief follow-up interview in which we asked the teacher about preparing for the lesson, what the major points of the lesson were, what surprises came up in the lesson, how to deal with different student ability levels in mathematics, and his or her philosophy of teaching mathematics. We used the data from both the lessons and interviews in our three-step analysis process, which we introduce in the next three subsections.

### Analyses

***Coding errors.*** In the first step in our analysis, we located and marked all student errors made during whole-class instruction. Errors were defined in two ways: (1) answers that were mathematically incorrect and (2) answers that the teacher treated as incorrect (e.g., Flevares, 2004). It was necessary to include the second part of our definition because, as Cobb et al. (1992) noted in their work on sociomathematical norms, classrooms have different standards for what counts as an error and what responses to an error are acceptable. We wanted to capture anything that the teacher viewed as an error in a classroom, and, consequently, the messages sent to students about what constitutes an error and an appropriate response to an error.

One coder marked errors in all 44 lessons. Another independent coder marked errors in 13 lessons, or approximately 30% of the total lessons. Simple agreement between coders for locating errors was .94, with a Cohen's kappa of .70. This is considered substantial reliability (Cohen, 1960; Landis & Koch, 1977). In addition, the two coders discussed and came to agreement on all discrepancies in coding.

***Coding teachers' responses.*** After locating and marking all errors, we coded teachers' responses to each error. We developed a coding scheme based on preliminary analysis of the data. This scheme consisted of two broad classes of teachers' responses, distinguished as CLASS 1 responses, which follow errors with teachers' statements, and CLASS 2 responses, which follow errors with teachers' questions. We further divided the responses into 11 more specific coding categories (listed below).

CLASS 1 responses, which were statements, included:

- Telling the student the answer is wrong,
- Giving the correct answer,
- Ignoring the error,
- Providing explanation or direction, and
- Students spontaneously correcting themselves.

CLASS 2 responses, which were questions, included:

- Re-asking the question,
- Clarifying the question,
- Asking for an addition to the answer,
- Asking for certainty or agreement,
- Redirecting the question, and
- Asking for student explanation.

Although we used these 11 categories to code the teachers' responses, we conducted analyses on both the 11 individual codes themselves and the two broad classes of teachers' responses. In addition, for the six CLASS 2 codes that are in the form of questions, we marked whether the teacher directed the response (i.e., the question) to the same student who made the error or to another student. (Please note that one code is technically not a teacher response. The code for students spontaneously correcting themselves captures the rare moments when a student made an error and quickly corrected himself, making a teacher response unnecessary.)

For consistency's sake, we coded only teachers' responses made in the teacher's turn immediately following the student error (Santagata, 2004). As with identifying errors, reliability for teacher-response coding was achieved between two coders. One coder marked the teachers' responses to errors in all 44 lessons. Another independent coder marked the teachers' responses to errors in 12 lessons, or approximately 27% of the lessons. Simple agreement between the coders was .80, with a Cohen's kappa of .77, which is considered substantial reliability (Cohen, 1960; Landis & Koch, 1977). As before, the two coders discussed all discrepancies between teacher response codes and came to agreement.

***Qualitative analysis.*** Literature on classroom discourse (e.g., Heath, 1982; Hiebert & Wearne, 1993) has made a strong case for augmenting and enhancing quantitative analyses of classroom conversations with a qualitative analysis or presentation of illustrative vignettes. In our final level of analysis, we looked for themes in teachers' interviews regarding teachers' beliefs about errors and searched the lessons for teachers' explicit comments about errors. We then chose examples of teachers' practices regarding errors that illustrated or conflicted with the teachers' statements about errors. These statements and practices were integrated into a presentation of four themes regarding beliefs and practices surrounding errors, with the goal being to examine how teachers' beliefs about the use of errors translated into inquiry-based practice regarding errors.

To begin, all statements teachers made about errors in their follow-up interviews were placed in one database. In their interviews, the Chinese teachers made 39 statements regarding errors, and the U.S. teachers made seven. We then added teachers' explicit statements about errors during each lesson to this database. We chose the statements from the lessons by searching each transcript for words such as "mistake," "error," "wrong," "incorrect," "inaccurate," and "problem." Only statements that truly dealt with errors as a concept were included. For instance, we included the statement, "Let's check them one by one to see if there are any *problems,*" whereas we did not include the statement, "Let's go on to the next *problem.*" In sum, the Chinese teachers made 46 explicit statements about errors in the lessons, and the U.S. teachers made 18 such statements.

After we placed all the statements in a single database, we coded each for themes regarding teachers' beliefs and ideas about errors (e.g., Lubienski, 2000a, 2000b). In all, we found 15 themes in these teachers' statements. Because many themes overlapped, and others were hardly mentioned, we integrated our findings into four major and recurring themes that we discuss in detail in the Results section.

After identifying the four themes from the teachers' statements about errors, we used them as a lens for looking at teachers' responses to errors in the lessons. Using the themes as our guide, we selected examples from the teachers' responses to errors that illustrated or conflicted with the themes in the teachers' statements about errors. We selected these excerpts of teachers' responses to errors after all the quantitative coding of teachers' responses had taken place. The excerpts augment our discussion of teachers' statements about errors and illustrate how some teachers' beliefs about errors played out in practice.

We should note that one primary purpose of this analysis was to find vignettes of teachers' practice and comments on practice that supplemented our quantitative findings. We believe that these vignettes are important because they can potentially reveal what teachers believed were the ideal ways to handle errors, the sociomathematical norms they believed they were creating, and, perhaps, widely held cultural beliefs.

## Results and Discussion

### Frequency of Errors in the Chinese and U.S. Lessons

To determine the frequency of errors in lessons taught in both countries, we first calculated whole-class time for each lesson, because this varied among lessons. We then divided the number of errors in each lesson by the amount of whole-class time in that lesson. The result provided us with a measure of the number of errors per minute of whole-class instruction. We then computed the mean number of errors per minute for all place-value lessons and all fractions lessons in each country.

For the place-value lessons, the mean number of errors per minute was .50 ($SD$ = .21) in the Chinese lessons and .40 ($SD$ = .35) for the U.S. lessons. For the fractions lessons, these numbers were .44 ($SD$ = .19) for the Chinese lessons and .55 ($SD$ = .25) for the U.S. lessons. We found no significant differences in the frequency of errors across countries for either the first-grade place-value lessons ($t[13] = .73$, $p = .48$) or the fourth- and fifth-grade fractions lessons ($t[27] = -1.33$, $p = .20$).

It is interesting that the Chinese method of dealing with errors, which Stevenson and Stigler (1992) and Wang and Murphy (2004) described, might suggest that Chinese teachers would foster an environment in which students feel free to make errors and to critique each others' errors. One might predict that students would make more errors in such an environment. Our results, however, did not support this conclusion.

### Teachers' Responses to Errors in Place-Value Lessons

Table 1 presents the prevalence of each type of teacher response, as well as each general CLASS of responses, for the Chinese and U.S. place-value and fractions lessons. For all the data in this table, we calculated the prevalence of each type of teacher response by adding the number of teachers' responses of that type in each lesson and then dividing by the total number of responses in that lesson. The percentage of responses of each type was averaged across all the lessons for each country. We then transformed these percentages, using the recommended arcsine transformation, so that we might use $t$-tests to draw inferences about statistical significance.

As Table 1 shows, teachers of place-value lessons from both countries more often asked questions (CLASS 2) than made statements (CLASS 1) about errors. However, Chinese teachers were significantly more likely to follow errors with questions than U.S. teachers, whereas U.S. teachers were significantly more likely to follow errors with statements. Although teachers from both countries primarily responded to errors during place-value lessons with questions, the Chinese teachers did so significantly more frequently. This finding is echoed in the fractions lessons and is discussed more fully in the next section.

What accounts for this difference? Of the 11 teacher response codes that comprise the two broad CLASSES of responses, significant differences between the place-value lessons from China and the United States only existed in four of those responses (see Table 1). The Chinese teachers of place-value lessons more frequently responded to errors by clarifying the original question or asking for an addition to the mistaken answer than the U.S. teachers of these lessons did. In contrast, the U.S. teachers asked for agreement with the mistaken answer more often than the Chinese teachers of place-value lessons and also provided more explanations. This may indicate that, although teachers of place-value lessons in both countries valued questioning students about errors, the U.S. teachers tended to question students by asking them to evaluate the answer. In contrast, the Chinese teachers questioned students by asking them to correct or explain the error.

Finally, in every instance in which the

TABLE 1. Mean Proportion of Each Teacher Response Type per Lesson, by Country

| | Place-Value Lessons | | | | | Fractions Lessons | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Chinese | | U.S. | | | Chinese | | U.S. | | |
| | *M* | *SD* | *M* | *SD* | *t*(13) | *M* | *SD* | *M* | *SD* | *t*(27) |
| Class 1: | .16 | .07 | .34 | .19 | 2.65* | .18 | .09 | .44 | .19 | 4.49** |
| Tell error is wrong | .03 | .05 | .1 | .16 | .92 | .03 | .04 | .13 | .07 | 4.49** |
| Give correct answer | .08 | .06 | .05 | .07 | .89 | .01 | .02 | .14 | .16 | 4.14** |
| Ignore the error | .02 | .04 | .05 | .07 | .77 | .005 | .02 | .02 | .03 | .94 |
| Student correction | .003 | .01 | 0 | 0 | .69 | .03 | .04 | .01 | .02 | 1.11 |
| Teacher explanation | .04 | .05 | .14 | .09 | 2.32* | .11 | .07 | .15 | .07 | 1.79 |
| Class 2: | .84 | .07 | .66 | .19 | 2.65* | .82 | .09 | .56 | .19 | 4.49** |
| Re-ask the question | .28 | .14 | .15 | .22 | 1.76 | .17 | .14 | .13 | .09 | .41 |
| Clarify the question | .11 | .07 | .03 | .06 | 2.76* | .02 | .04 | .05 | .05 | 1.73 |
| Ask for an addition | .09 | .04 | 0 | 0 | 5.57** | .11 | .12 | .04 | .05 | 1.91 |
| Ask for agreement | .11 | .05 | .25 | .11 | 3.33* | .18 | .09 | .13 | .12 | 1.39 |
| Redirect discussion | .22 | .15 | .24 | .23 | .28 | .24 | .14 | .14 | .11 | 2.21* |
| Ask for explanation | .03 | .05 | 0 | 0 | 1.61 | .09 | .09 | .06 | .07 | .93 |

*$p < .05$.
**$p < .001$

teacher followed up an error with a question about the error, we noted whether the teacher addressed the follow-up question(s) to the same or different student(s). We found that teachers from both countries tended to follow up with the same student who made the error during the place-value lessons. In the Chinese lessons, teachers directed .66 of these responses to the student who made the error ($SD = .17$); in the U.S. lessons, teachers directed .77 of these responses to the same student ($SD = .14$). The difference between the countries was not significant, $t(13) = 1.26$, $p = .23$.

### Teachers' Responses to Errors in Fractions Lessons

Results for the teachers' responses in the fractions lessons were similar to the results from the place-value lessons (see Table 1). Again, teachers from both countries favored asking questions rather than making statements to students following an error. However, Chinese teachers were significantly more likely to respond to an error with a follow-up question than U.S. teachers, whereas U.S. teachers were significantly more likely to respond with a statement about the error than Chinese teachers.

Interestingly, the difference between U.S. and Chinese teachers in the fractions lessons, in terms of asking questions or making statements, was even more pronounced than in the place-value lessons. Although we hypothesized that the Chinese teachers would generally question their students more about errors than U.S. teachers, we also expected that the difference would be less pronounced for older students. We thought teachers would assume that older students would be more prepared to deal with complex questions. These data provide no evidence to support this hypothesis.

Although teachers' use of each CLASS of response was similar for the place-value and fractions lessons, different categories of responses accounted for the use of each CLASS of response in the fractions lessons. In particular, the U.S. teachers of fractions lessons told the student the answer was wrong or gave the correct answer significantly more often than the Chinese teachers. For these U.S. teachers, this finding indicates a move from correcting the error indirectly (through explanation of the error or asking for agreement with the error, as in the place-value lessons) to more immediately correcting the error. In contrast, the Chinese teachers of fractions lessons redirected the discussion about the error, indicating a commitment to having students work through their errors, more often than the U.S. teachers. Surprisingly, as we found in the place-value lessons, the Chinese and U.S. teachers of fractions lessons tended not to ask for student explanation and instead used other methods to foster student thinking about errors.

Finally, as with the place-value lessons, we examined whether teachers of fractions lessons directed follow-up questions to the same student who made the error or to another student(s). The Chinese teachers directed .52 of these questions to the same student who made the error ($SD$ = .20), whereas U.S. teachers directed .55 of these questions to the same student ($SD$ = .30). Again, there was no significant difference across the countries, $t(27) = .37$, $p = .72$. However, teachers from both countries clearly were more likely to involve other students in questions about an error in the fractions lessons than they were in the place-value lessons.

### Qualitative Analysis of Teachers' Practices and Beliefs

In this section, we present four themes derived from teachers' statements about errors and excerpts from lessons that illustrate these statements. The themes are (1) the emphasis on creating an environment in which students feel free to make errors; (2) the use of common errors, or "good mistakes," in instruction; (3) the belief in review as a method for dealing with errors; and (4) the importance of asking students to think through their own errors. As we report on these themes, we draw conclusions about what they might mean in terms of fostering inquiry surrounding errors in mathematics instruction.

***A classroom environment that supports errors.*** As mentioned in the introduction, associationist theories in mathematics have treated errors as threats that at worst could confuse students and at best were a sign of not practicing number facts. Despite this associationist "fear" of errors, both U.S. and Chinese teachers espoused the belief that errors are natural and helpful in discovering what students need to learn. In fact, 31 teacher statements about errors in the interviews and in the lessons endorsed one of the following ideas: (1) errors are acceptable and students should speak freely without fear of making an error, (2) errors help teachers measure what students know, and (3) it is important to detect errors. Some of the statements teachers made about this follow:

> U.S. teacher: Usually if we have a trouble in math, I can figure out what your problem is and we can work on that one little part and then we can go.
>
> Chinese teacher: I won't discourage [the children who make mistakes] and will let them speak out their ideas confidently. It doesn't matter if you say it wrong. If only you dare to say it, you're so great. In this way all the students can fully express themselves, and their problems can be exposed and resolved in a timely manner. If they have success, I'll praise and encourage them right away in class and get them inspired.

Clearly, many teachers from both countries saw the virtues of errors for monitoring student understanding. However, what was less clear was whether the teachers imparted this belief to students. In this regard, we found that the Chinese teachers were more open with their students than the U.S. teachers about the freedom to make errors. These teachers made four comments in class that alerted students that they should feel free to make errors; U.S. teachers made no such statements. Statements such as the following (from Chinese classrooms) informed students that they should not be embarrassed to make errors:

> Teacher: Who will talk about 14? What is 14 composed of? It doesn't matter if you're wrong. You, please.
>
> Teacher: Ting, do you understand now? I think you must be mistaken. Come on and tell us why. It doesn't matter.

In addition, an intriguing example from a Chinese teacher's lesson illustrated an attempt to foster a sociomathematical norm in the classroom in which students did not judge others for their mistakes and allowed students to correct their own mistakes. The excerpt provides a glimpse of a teacher asking her class to decide if a student who made an error deserves a five-pointed star. Chinese teachers of place-value lessons gave stars and red flowers to students who performed well. In this example, the students did not want to award the mistaken student a star, but the teacher had other ideas:

> Teacher: What is the number one before 19? He's quick.
> Student 1: The number one before 19 is 20. [*error*]
> Teacher: Is there different opinion? [*ask for agreement*]
> Students: [in unison] Yes.
> Teacher: Can I give him this five-pointed star?
> Students: [in unison] No.
> Teacher: He said the number one before 19 is 20. Want to change it?
> Student 1: I want to change.
> Teacher: Say it again.
> Student 1: The number before 19 is 18.
> Teacher: Okay, he changed it to the correct answer. Let's give him a five-pointed star, okay?
> Students: [in unison] No.
> Teacher: He changed his mistake, so we can also give him a star. [The teacher gave him a star for changing his answer.]

The example shows that the Chinese teacher was not pleased with the other students' opinion that the student should not get a star. In the interview, this teacher commented on what she perceived to be the students' poor attitude towards making mistakes: "Sometimes I feel some children lack understanding for other students. I shall tell them they should understand each other and allow others to correct their mistakes. You see some students said it wrong at first and then corrected it. But some other students still thought they should not be awarded red flowers. I should develop in them these ideas by any means possible in our daily lesson." From this example and from other statements teachers made about the importance of students making errors, we inferred that many teachers placed a premium on creating a "risk-free" environment for students to make errors (e.g., Lampert, 1992). The teachers created such an environment not only by believing in the use of errors as resources but also by reminding their students that they should feel free to make errors themselves and should allow others to make errors. In our opinion, this practice can be pivotal for fostering a risk-free environment in which students feel free to make errors and therefore feel free to inquire about these errors.

***Getting to the "good mistakes."*** Some teachers indicated that they planned instruction so that students would make certain errors that are particularly useful in instruction. For instance, two Chinese teachers said in their interviews that they intentionally planned instruction to "set up" their students to make certain errors so that they could discuss these common errors. One Chinese teacher described how she used questions in class to try to get students to say the wrong answer but found that the students would not be fooled: "I planned a step today which is designed to let students guess what's one-half plus one-third. According to my previous teaching experience, some students may say it's two fifths. But today, to my surprise, they all answered 'five-sixths' with one voice."

We did not find evidence from U.S. teachers' interviews of deliberate planning to elicit mistakes. However, we did find evidence of a related strategy. Some U.S. teachers distinguished between "good mistakes" and "bad mistakes"; in these cases, good mistakes were those that brought possible common mistakes to light. The following is an example of a U.S. teacher working with a "good mistake" in a fractions lesson:

> Teacher: Yes, how would you write one whole? [The teacher was looking for the answer 1, not sixteen-sixteenths.]

Students: [shouting out] Just put one! Sixteen-sixteenths! [*error*]

Teacher: Can't do sixteen-sixteenths. [*tell the student the answer is wrong*] I guess you could, but sixteen-sixteenths . . . Jason, look at me. I'd like to say something. If we did sixteen-sixteenths, Maria . . . [*teacher explanation*]

Student: Yeah.

Teacher: That would indicate that there are 16 pieces.

Student: Oh.

Teacher: And there are 16 parts. There are 16 in the whole and there are 16 parts. Exactly. So good point, that was a good mistake. Okay, but how would we write one whole?

Although only five U.S. teachers made explicit statements in their lessons about planning instruction so that students made certain "good mistakes," we believe this is an intriguing instructional practice for promoting inquiry into errors precisely because its use indicates common mistakes students make or elucidates aspects of the material that might otherwise go unnoticed. Likewise, the two Chinese teachers' method of planning to elicit mistakes is particularly compelling. In both examples, the teachers built instruction around the mistakes. We suggest, then, that another way to foster inquiry in discourse around errors is planning instructional practices for common errors.

***Review, review, and review again.*** The teachers made 13 statements about the power of review to help students make fewer mistakes and remember correct answers, including these two excerpts from the interviews:

Chinese teacher: We have a remedial class to help these students [who struggle with material and make mistakes]. In the remedial class, the teacher will repeat the key points, correct their mistakes, and help them to solve their learning problems. When they have touched the knowledge several times, they have a reinforced memory, and their ability is improved on this basis.

U.S. teacher: There [was] a girl . . . who had just sort of appeared to have forgotten everything. I mean she was, like, adding and multiplying and this and that and didn't know what the numerator . . . and so we turned it into something sort of funny, but I thought, okay this would be a really good chance to notice that she needs to review all sorts of things.

The importance of review was embodied in the Chinese lessons in an intriguing way. In many of these lessons, when a student made an error, the teacher got that student to remain standing while other students provided the correct answer. After the correct answer had been stated, the teacher had the student who originally made the error repeat the correct answer. The following example illustrates this practice in a fractions lesson:

Teacher: Who will talk about the meaning of the rationale of consistent quotient? Fan?

Student 1: Two such numbers divided or multiplied . . . [*error*]

Teacher: Who will help her? Xiaobin. [*re-ask the question*] [Student 1, Fan, remains standing while Student 2, Xiaobin, answers the question.]

Student 2: If the two numbers are multiplied or divided by the same number, the quotient is unchanged. [*error*]

Teacher: What are the two numbers? [*ask for addition to answer*]

Student 2: The dividing number and the divided number. If the dividing number and the divided number are multiplied or divided by the same number, the quotient is unchanged.

Teacher: Right?

Students: [in unison] Yes.

Teacher: Say it again. [The teacher points to Student 1 to repeat the correct answer that Student 2 had given.]

Student 1: If the dividing number and the divided number are multiplied or divided by the same number, the quotient is unchanged.

In an interview after this lesson, the teacher talked about the practice of asking the student who made an error to remain standing until she could repeat the correct answer.

Interviewer: Oh, I notice a situation in the class that you'll ask the students who

> didn't give the correct answer to repeat it again after the correct answer was given.
>
> Teacher: Yes, if I get them consolidated like this they will have a right concept in their mind instead of the original wrong one. No matter whether or not they can remember it, at least they will have the impression first.

This teacher could almost have been quoting associationist theory as she discussed the idea that students should be left with the "right concept" instead of the "original wrong one." The quotes in this section show that both Chinese and U.S. teachers emphasized review strongly as a method for dealing with errors.

***Students working through errors.*** In our quantitative analysis of the teachers' responses to errors, we found that the Chinese teachers responded to errors more frequently with questions than the U.S. teachers, and the U.S. teachers responded more frequently with statements than the Chinese teachers. We concluded that the emphasis on questions by the Chinese teachers was a matter of encouraging students to work through their errors rather than correcting their errors immediately. In support of this practice, the Chinese teachers made nine explicit statements about allowing students to work through their errors, compared to only three by U.S. teachers. The following statement regarding the importance of letting students work through their errors is from a Chinese teacher: "When doing exercises about fractions' basic characteristics, if a student doesn't multiply or reduce the numerator and denominator simultaneously, it shows that he doesn't really understand 'simultaneously.' We need to get him to understand that he must change the numerator and denominator simultaneously or the quotient will not keep invariable. So we should let the students do some exercises and find their problems through it. They still won't understand if we just explain it again and again."

We found one U.S. teacher who at times used discourse surrounding errors in which she withheld judgment about answers and gave students the authority to resolve the error. This teacher, while teaching a fractions lesson, not only allowed students to call the error into question but also let the student who made the error seek explanation from her classmates:

> Teacher: You want to go ahead and finish us up? [The teacher wants Student 1 to answer the last question on the board.]
>
> Student 1: 13.6 [*error*]
>
> Teacher: 13.6? [*ask for agreement with answer*]
>
> Students: [several students calling out] I thought it was 13.06. Yeah, 06.
>
> Teacher: Uh-oh . . . they don't like your answer.
>
> Student: [calling out] I don't either.
>
> Teacher: Who can help her out and tell her why? [points to Student 2] Thank you for being quiet when you raised your hand.
>
> Student 2: Um, without the 0 it would be six-tenths, and if you put the 0 it makes it six-hundredths.
>
> Teacher: Do you understand that?
>
> Student 1: Kind of . . .
>
> Teacher: Do you need a little more help explaining it?
>
> Student 1: It's kind of confusing.
>
> Teacher: It's kind of confusing. Brian. Go ahead now, try to explain it to her.
>
> Student 3: Um, you already have a 13, but if you put 13.6, it would be 13 and six-tenths, and it says there are 13 and 6 one-hundredths, not tenths. And if you didn't put the zero it makes it tenths.
>
> Teacher: Does that help? Does anybody else have a way they think about it? [pauses] Stephanie, when we think about the tenths and the hundredths, you think about money, and so 6 pennies is a lot different than 6 dimes, or 60 cents. So when it is six-hundredths that means it is 6 out of every hundred, or 6 pennies. And that is why it is very, very important to get it stuck over in the pennies slot, or the hundredths slot. Does that help?
>
> Student 1: Uh-huh.

Although there are no guarantees that the student who originally erred understood what she had done wrong, at the very least, this student was provided with several examples that could help clarify the concept for her, thus promoting additional understanding.

Another example from this teacher's lesson focused on allowing a student who made a mistake to work through her thoughts and then seek help from her classmate:

> Teacher: What number is actually in the dollar?
> Student 1: The 5. (*error*)
> Teacher: She says the 5. Now are you over here looking at the money, $368.27? Okay, you're confusing me girl. [*clarify the question*]
> Student 1: The 8.
> Teacher: The 8, okay. So the 8 is in the dollar slot. What are we going to round it to?
> Student 1: I look up there at the . . . (*error*)
> Teacher: Keep going, keep working through so I can help you. [*ask for student explanation*] We're looking at rounding $368.27 to the nearest dollar, and she's stuck. Let me know if you need some help and we'll help you. [*re-ask the question*]
> Student 1: I need help. [*error*]
> Teacher: Help. Okay, Kathleen, help her out. [*re-ask the question*]
> Student 2: You look at the 2 and it's less than 5 so the 8 and everything behind it turns to a zero.

The examples presented in this section show that some Chinese teachers expressed an interest in allowing students to work through their errors. This interest was also reflected in their use of CLASS 2 responses, or questions that prompted students to think through their errors. Although we found significantly fewer indications that U.S. teachers prompted students to think through their errors, examples like the last two illustrate that U.S. teachers can implement such practices. Based on these examples, we suggest that teachers foster inquiry in their responses to errors by allowing the student who made the error or other students to notice and correct it.

## Conclusion

In this section we suggest four ideas regarding how teachers might use errors during mathematics instruction to promote an inquiry environment. In doing so, we also consider what may be lacking in some of these approaches.

The first idea we culled from our data is that, before teachers use errors in instruction, they often create a classroom environment in which students feel free to make errors and in which students believe it is common to notice and discuss errors. In these classrooms, students are comfortable making an error and do not mind if other students correct it or evaluate it. We found that some teachers sought to create such an environment by telling the students not to feel afraid of making errors and by acknowledging "good mistakes" that are useful in instruction. Without this environment, students may miss a critical aspect of schooling: students come to school because they do not know everything and, by uncovering and then correcting their errors, they will learn.

In contrast, the literature on mathematics classrooms (e.g., Santagata, 2004; Stigler & Perry, 1988), especially U.S. classrooms, provides multiple examples of teachers discouraging, hiding, or prohibiting errors. In these classrooms, the assumed, intended atmosphere seems to be telling students, "You're so smart, you don't even make any mistakes!" This sort of atmosphere, in turn, should lead students to feel good about themselves and about how smart they are in mathematics. Although teachers may have good intentions when fostering such an environment, this atmosphere could lead students to become afraid to make mistakes for fear of being seen as stupid. Indeed, several teachers in our sample endorsed this position, such as the Chinese teacher who said, "Teachers should put more simple and easy-answered questions to the weak to give them more chances to show themselves. Therefore they can improve their sense of achievement." Although this teacher (and several others) endorsed posing simpler questions to lower-ability students, presumably so they would not make errors or could at the very least attempt to solve the problems, other evidence from the Chinese lessons indicates that teachers expect and encourage errors. Our conclusion from observing classrooms that foster an environment in which errors are commonplace—and some are even good—is that students' self-esteem does not suffer and they are better able to correct mistakes and learn more mathematics than when errors are discouraged.

The second idea for using errors to promote inquiry is that teachers pushed students to think about their errors but also showed that this is not necessarily accomplished only by asking for student explanation of errors. Chinese teachers in particular emphasized asking questions to respond to errors. Asking for student explanation was only a small part of the questioning process in both the Chinese and U.S. lessons. We believe that asking questions is an important tool for fostering inquiry because it forces students to recognize and attend to the process of correcting an error and, sometimes, the reasons behind an error. Only by asking students questions about errors can teachers create the kind of inquiry environment envisioned by researchers like Borasi (1994) and Kazemi (1998), and by the NCTM (1991). By varying the types of questions, and not relying solely on questions that request explanations, teachers give students the opportunity to become aware of multiple ways of examining a mathematical problem and to potentially engage with the problem at different levels.

We take from these results that asking "why" questions is not the only way to query students successfully about an error. Many of the CLASS 2 responses, such as asking students whether they agreed with the error, or redirecting students to think about the original question in a different way, may be equally valuable. In fact, we saw instances in which "why" questions were confusing and did not help students attend to an error as well as redirecting the discussion did. We hope that the vignettes we provided can serve as examples of ways to begin inquiry surrounding errors.

The third idea from our data is that mathematics teachers, and U.S. teachers in particular, still seem to be driven by remnants of associationist theory. The finding that U.S. teachers felt compelled to make more statements about errors than their Chinese counterparts is not surprising in light of the strong effect of the work of Thorndike and other behavioral psychologists (e.g., Skinner) on mathematics education. Although we do not wish to argue that associationist perspectives are necessarily negative, particularly when it comes to the necessity to practice some basic mathematical skills, we believe the teacher can strike a balance between letting the student know he or she has made an error and being overly concerned with correcting the error immediately. In general, literature on discourse in mathematics classrooms (e.g., Borasi, 1994; Kazemi, 1998; NCTM, 1991) suggests that teachers should correct an error only after giving students an opportunity to notice and correct the error themselves. Obviously there is a fine line between giving students time to correct their errors and letting the discussion about an error go on too long (Ball, 1993). In our qualitative analysis of teachers using review and other methods for inquiry around errors, we found that teachers can think carefully about giving students the opportunity to work through their errors but can also ensure that students leave the conversation with the correct answer and process in mind. In our view, the Chinese teachers provide an excellent example of such a balance.

The fourth idea about how teachers might use errors is that teachers can anticipate student errors as they plan their lessons, thinking carefully about how they will respond to these errors (e.g., Fennema et al., 1993, 1996). "Bug analysis" (e.g., Brown &

Burton, 1978; Brown & VanLehn, 1980; VanLehn, 1990) preceded this idea in that it emphasized analyzing student mistakes and planning responses to these mistakes. However, "bug analysis" typically cast errors as problems to be diagnosed and eradicated. The teachers whom we quoted in our qualitative analyses expanded on this idea, emphasizing the importance of considering how errors can be used to enhance students' thinking about a concept or to help students consider why common errors "make sense." In other words, if teachers want to use errors as "springboards for inquiry," they have to be ready to jump.

A word of caution is in order, however. We should note that in this study we report what we saw in classes and cannot necessarily link the practices we saw to "successful" use of errors. We believe that our findings point toward some methods for using errors for inquiry in mathematics classrooms, and indeed such inquiry environments have been linked to improved student performance (e.g., Kazemi, 1998). However, we did not see such high-level discourse practices surrounding errors as students debating each other about the plausibility of an error, as the NCTM (1991) and Ball (1993) proposed. In addition, the focus on discourse in this article did not allow us to look at how teachers might have used gestures or learning tools to help students examine errors. Thus, this article is only a beginning toward looking at practices in different countries that may help teachers use errors more effectively.

In sum, we found that, although errors were as common—and, presumably, as welcome—in U.S. classrooms as in Chinese classrooms, teachers' responses to these errors followed interesting, but different, patterns. We expect that, as researchers learn more about how teachers can use errors to promote inquiry in classroom discourse, teachers and students can maneuver more successfully with student mistakes. Ultimately, we hope that both teachers and students do not fear making mistakes and come to see them as opportunities to learn.

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