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Explanations of Mathematical Concepts in Japanese, Chinese, and U.S. First- and Fifth-Grade Classrooms

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In this research, I examine some of the classroom processes that may be responsible for the stellar mathematical performance among Asian children compared to U.S. children. The study documents differences in the frequency and type of mathematical explanations during lessons observed in 80 U.S., 40 Chinese, and 40 Japanese 1st- and 5th-grade classrooms. Explanations occurred more frequently in the Japanese and Chinese classrooms than in U.S. classrooms. Furthermore, typical explanations in the Asian classrooms were more substantive than those in the U.S. lessons, and Japanese children were learning about more complex topics than their peers in Taiwan or the United States.

Many research reports, as well as popular news reports, have informed us that Asian children outperform U.S. children in almost every academic subject, especially in mathematics (e.g., Applebome, 1996; Beatty, 1997; Stedman, 1997; Stigler & Hiebert, 1997). Differences in achievement between Asians and Americans have been found to exist as early as kindergarten and to be dramatic by the time children reach fifth grade (Song & Ginsburg, 1987; Stevenson et al., 1990; Stevenson, Lee, & Stigler, 1986). As an example of some of the remarkable findings, Stevenson et al. (1986) reported that by the fifth grade the highest scoring of 20 American classrooms did not perform as well as the lowest scoring Japanese classroom and outperformed only 1 of 20 classrooms in Taiwan.

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Although it is apparent that Asian children are outperforming U.S. children in mathematics, the causes of such superior performance deserve further explication. A great deal of the extant research (e.g., Chen & Uttal, 1988; Stevenson et al., 1990) has focused on Japanese and Chinese children, especially on social and cultural supports for exceptional academic performance, such as the ways in which parents expect and support their children's high academic achievement. Some recent reports (e.g., Fennema, Franke, Carpenter, & Carey, 1993; Perry, VanderStoep, & Yu, 1993; Stigler & Fernandez, 1995; Stigler, Fernandez, & Yoshida, 1996; Stigler & Hiebert, 1997; Stigler, Lee, & Stevenson, 1987; Stigler & Perry, 1988b) have focused on classroom processes for some of the more proximal causes of the achievement differences and have presented detailed analysis of classroom interactions that may lead to the transformation of mathematical understanding. My investigation follows this line of research and relies on the assumption that it is important to look in classrooms for potential contributors to cross-national differences because it is difficult to conceive of most children performing well in mathematics without classroom support. In other words, good instruction should make a difference in children's developing understanding of mathematical concepts.

One obvious place to look for the source of the reported academic differences is in the way that mathematics instruction is delivered in the United States versus Taiwan and Japan. As Mayer, Sims, and Tajika (1995) put it, "cross-national differences in mathematics achievement are related to differences in the quantity and quality of mathematics instruction" (p. 444). Mayer et al. also pointed out that meaningful explanation promotes problem-solving competence. Mathematical explanations in Asian and U.S. classrooms were chosen for careful analysis primarily because important information about mathematical concepts is communicated through this discursive form and thus are expected to have an impact on the development of mathematical knowledge. Indeed, explanations have not only been cited as a critically important form of mathematical discourse (e.g., Leinhardt, 1988) but also as a forum for socializing and becoming socialized into a particular educational culture (Mehan, 1982).

It has also been shown that in U.S. classrooms—especially in mathematics instruction—almost all new content is introduced through teacher explanation (e.g., Barr, 1988; Stodolsky, 1988). Without the introduction of new material, and the substantive description of how this fits in with the focus of the lesson, we probably would expect little from schooling. In other words, one of the primary purposes of educational institutions is to convey material that is new to students, and this is done often, although not exclusively (e.g., Ma, 1999; Stigler et al., 1996; Stigler & Hiebert, 1999), through the practice of providing verbal explanations.

Another reason for focusing on mathematical explanations is that considerable attention is devoted to mathematical explanation in the newly developed U.S. mathematics curricula, even those that rely heavily on manipulatives (e.g., Wagreich et al., 1997). These new curricula are being adopted widely and were produced in the spirit of the National Council of Teachers of Mathematics' (1989, 1991) *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics*. Because explanations have remained an essential component of mathematics curricula, explanations in mathematics classes were seen as an important discursive form to analyze and understand.

Thus, this study examines how mathematical ideas are explained in first- and fifth-grade classrooms in the United States, Japan, and Taiwan. The goals are to document the differences in the frequencies and types of explanations teachers provide and to discuss the impact of these differences on children's opportunities to develop mathematical knowledge.

METHOD

Background

Most of the children observed as a part of this investigation also participated in a battery of tests designed to measure mathematical understanding. A total of 5,524 children across 160 first- and fifth-grade classrooms were tested on a wide range of skills including computation, word problems, concepts, estimation, mental image transformation, mental calculation, graphing, and measurement. The results of these tests have been presented in detail elsewhere (Stigler, Lee, & Stevenson, 1990). The outcome at the most general level reported by Stigler et al. was that, in the first grade, the Japanese children did better than children from the other sites on almost every test, and the children from Taiwan tended to do better than the children from the United States. By the fifth grade, however, the children from Taiwan had clearly surpassed their U.S. peers and approached the level of performance attained by the children from Japan on almost every subtest of mathematical knowledge.

Sample

Classrooms were observed in Taipei, Taiwan; Sendai, Japan; and Cook County, Illinois (including Chicago) in the United States. In both of the Asian cities, 10 schools participated, whereas in the United States, 20 schools participated. A larger number of U.S. schools was used because classrooms in the United States were more diverse ethnically and linguistically, and it was important both to capture the diversity and to avoid having an overly biased sample.

The variables on which the U.S. schools provided diversity included (a) type of school (public or private); (b) location of school (inner city or suburban); (c) ethnicity of students (majority White, majority African American, majority Latino–Latina, or multiple ethnicities represented in the school); (d) native language of most of the students (English, Spanish, or other); and (e) socioeconomic levels. Because of the widespread diversity on so many variables in the U.S. sample, it was rare that the sample could be meaningfully divided to give a reliable picture of a subsample. The exception was the set of factors we controlled for when selecting our sample: whether the school was public or private and, if public, whether the school was located in or outside the city of Chicago. Of the 20 U.S. schools selected for this investigation, we had intentionally chosen 9 Chicago public schools, 6 suburban public schools, and 5 private schools (all Catholic schools, 3 in Chicago and 2 in the suburbs). These demographics were representative of the school children in Cook County. Furthermore, the dimensions of public versus private and inner city versus suburban are implicated in differences in the quality of education (e.g., Bryk, 1993); thus, we wanted to ensure that none of these major groups were left out. By contrast, the Japanese and Chinese schools were relatively homogenous with respect to these factors, with the exception that the Chinese schools represented a larger range of socioeconomic levels than the Japanese schools.

In each of the schools, two first-grade and two fifth-grade classrooms were observed, four times each. This yielded 160 observations in Sendai, that is, 10 (schools) × 2 (grades) × 2 (teachers in each grade) × 4 (observations for each teacher), 158 in Taipei, and 298 in Cook County (because 20 schools were used), for a total of 617 observed lessons in the three countries.[1]

All of the classroom observations were conducted during the second semester of the school year. In most cases, the observations took place during the middle and end of the second semester. Deviations from this generalization were caused by scheduling difficulties (e.g., avoiding weeks when standardized testing was taking place). The observations were conducted relatively late in the school year so that teachers and students were familiar with each other and had developed consistent patterns of interaction.

Procedure

The observers took records in several schools, in one of the three countries. Some of the observers from each of the three sites were brought together for initial training, so that agreement about what was to be observed was reached before the actual observations took place. This information was used to train the remainder of the observers in the three countries who initially were not gathered together. Each observer was native to the country in which he or she was observing, with one exception (one of the U.S. observers was native to England). For the most part, the observers were graduate students in departments of education or former teachers.

[1]Although 640 observations were planned, not all were conducted due to prolonged teacher illness or to several U.S. fifth-grade teachers being responsible for teaching both fifth-grade classrooms in the school.

Thus, most observers had prior experience with the rapid pace of the mathematical discourse that typically took place in the classrooms. Most of the observers were women.

Observers were instructed to write down as much as possible during the entire mathematics class. The observers were trained to record the ongoing flow of behaviors and to include descriptions of the lesson, including the materials used and the specific nature of the content. They paid special attention to the verbal remarks made by teachers and students; they had been instructed to include as many of these remarks as possible in their notes. Also, the start of every minute was noted in the running commentary. These 1-min intervals were included so that it was possible to estimate the duration of various activities. None of the observers were aware of any of the specific goals of this study.

The narrative observational notes were read and summarized into English by two coders who were monolingual, bilingual, or trilingual in English, Chinese, or Japanese. The observation notes were coded at two levels. The first level involved dividing whole lessons into meaningful lesson segments. Each of these segments was coded for topic, material, and activity. A new segment was coded each time any of these aspects (topic, material, or activity) changed within a lesson. Each of these three aspects was defined at a relatively molar level. For example, *topics* included things such as measurement and adding fractions. *Materials* included items such as textbooks, blackboards, and worksheets. *Activities* included explanations, evaluations, and seatwork, among others.

Explanations, the focus this investigation, were coded in the lessons as an activity. Thus, it is important to characterize just how explanations, when they constituted the focus of the classroom activity, were identified within the narrative observational notes. Thirteen mutually exclusive categories of activities were defined in a coding manual for the first-grade lessons, and two additional activities were added for coding the fifth-grade lessons. The coding manual included criteria for coding each of the activities and was accompanied by a set of accumulated examples for making decisions about what activity was taking place. For example, the criterion for coding an activity as an explanation activity was, "The teacher or student explains. This includes explanation of how to do something and/or of why to do something. The specific explanation (or at least the type of explanation) should be included in the summary of the segment." Each activity was explicitly contrasted with other activities such as demonstrating or giving directions. In this way, the coders were made aware of the importance of choosing the best-fitting category for any activity and were also attuned to the fact that activities that were coded as explanations required justification in terms of providing the explanation, or at least the type of explanation, in the summary notes. Because it was possible that more than one activity could take place within the span of 1 min (the smallest unit of time for which segments were created), the predominant activity within that minute was coded as the activity for that segment. Explanations were further categorized by type of explanation. However this was done in a later phase of the research and is described in the Results section.

The second level of coding involved constructing summaries (in English) of what was going on in the class during each segment. Most of the analyses reported here are based on the summaries produced from the classroom transcripts. As Stigler and Perry (1988b) reported, the decision to rely on summaries of the observations was made to deal with inconsistencies in the amount of detail the observers recorded. The summaries were constructed both to capture the flow of the lesson and to recapitulate in detail what was going on during the lesson. Standardized procedures were outlined so that certain things would always be included in the summary, if they occurred during the lesson. This was important because only one activity was coded for any one segment, although other minor activities could occur during this same time. For example, and directly related to the purpose of this investigation, all explanations were incorporated into the summary verbatim (if the quote was available in the notes, otherwise a description of the explanation was incorporated into the summary), even if the purpose of the activity was something other than providing explanations. Thus, explanations could occur at two levels: at the level of the activity, in which the majority of the segment was devoted to providing explanations, and an explanation provided in the service of some other activity, embedded within an activity that was not centered on providing explanations.

To achieve reliability in the initial coding of the lessons, all lessons ($N = 617$) were coded by two coders. Agreement between coders on segmentation of lessons and on the topic, material, and activity coded for each lesson was quite high (Cohen's kappas exceeding .77 and simple agreement exceeding 85% on segmenting the lessons, on the topic of the segment, on the materials used in the segment, on the activity occurring within the segment, and on information to be noted in the summary), with the exception of one coder, whose original coding was redone by the most reliable coder. Any disagreement between the two coders was resolved by discussion among the entire group of coders ($N = 6$). Further detail about how the summaries were constructed is reported by Stigler and Perry (1988a, 1988b).

Coding Explanations

All explanations were extracted from the observational notes and coded as (a) a separate extended activity lasting at least 1 min or (b) a passing event embedded within a segment devoted to some activity other than explaining. Note that the predominant activity within any minute was the activity coded for that minute. Also note that an explanation appearing in the observation notes, even if it occurred during an activity other than explanation, was included in the summary. With this approach, the difference between extended explanations and briefer explanatory information could be captured.

It was also important to code the topic being explained for each explanation. Because not all topics receive the same sorts of explanations, we examined explanations by topic, for example, examining explanations of how to add fractions separately from explanations of why place value is important for addition of multidigit numbers. Thus, for each explanation, a topic was noted.

Finally, distinctions were made within topics for different types of explanations. This sort of coding was done only for the most frequently occurring topic in all three countries, at each grade level. Coding and analyses related to explanations of the most frequently occurring topics is discussed in detail in the Results section.

RESULTS

Explanations in the first and fifth grades were examined separately because there was no reason to expect comparability between the two grades. For each grade, a general description of the observations is presented to give a context for interpreting the results concerning explanations. Next, general results about the frequency and duration of explanations are reported. Finally, results about specific types of explanations are reported.

FIRST GRADE

Description of Classroom Activities as Contexts for Explanations

The most commonly occurring activities in the first grade were question-and-answer periods, seatwork, or question-and-answer periods intermingled with seatwork (see Table 1). Across all three observation sites, these accounted for 63% of all 1,597 segments (61% of all segments in Japan, 65% in Taiwan, and 64% in the United States). Teachers' explanations frequently occurred during these activities—67% of explanations that were present in the first-grade data ($N = 432$) were found in segments devoted to these activities, although very few ($n = 16$) occurred in segments devoted to seatwork alone. What this suggests is that teachers tended to provide explanations when they received cues from students that they did not fully understand a concept (e.g., in question–answer periods) and as they were guiding students through seatwork. The recording of few explanations during seatwork-only segments may have happened because, indeed, few of these interactions occurred. Alternatively, explanations during seatwork-only segments may have been missed by the observers because explanations delivered to individual children were inaccessible to the observers. Likewise, explanations provided by peers typically could not be heard by the observers. The remainder of all explanations tended to occur in

TABLE 1
Activity Frequencies in First-Grade Classrooms

	Occurrence (n)			*Percentage of Activity That Included Explanation (%)*		
Activity	*Japan*	*Taiwan*	*United States*	*Japan*	*Taiwan*	*United States*
Question-and-answer	134	139	272	56	41	27
Seatwork (SW)	84	57	136	7	0	7
Question-and-answer with SW	57	41	90	68	24	21
Explanation	73	1	22	100	100	100
Evaluation	51	28	58	31	21	12
Choral responses	16	3	43	25	0	7
Teacher gives directions	10	4	28	20	0	21
Mental calculation	0	45	0	—	4	—
Other	28	46	131	0	0	0
Total	453	364	780	47	21	18

segments devoted to explaining or evaluating student work (i.e., 87% of the remaining explanations were found in explanation or evaluation segments).

Analyses were conducted separately for extended explanations and brief explanations that were embedded in other activities because it was reasoned that extended explanations were likely to be more important than brief explanations. The rationale for this was that, during extended explanations, children would have an opportunity to build mathematical knowledge and to appreciate the explanation if it lasted longer than 1 min and was not embedded within some other activity. This follows closely from what Stigler and his colleagues (e.g., Stigler et al., 1996) suggested: The chance that mathematical content is processed increases as more time is spent on that concept because students "are allowed more time to construct meaning for the mathematics that goes on around them" (p. 162). Toward this end, the number of lessons (out of the four lessons that were observed) that contained at least one extended mathematical explanation, by a teacher or by students, was compared across countries.

Although the impact of extended versus brief explanations might be different, the contents of the explanations at the two levels were quite comparable. In other words, teachers tended to convey similar information in both extended and brief explanations. For example, in one Japanese lesson, the students were provided with an extended explanation about adding. This segment was summarized as follows:

> The teacher explains the importance of the methods used when adding and says: "The answer is 41, but that is not as important as the method by which you get it. The crucial thing is the right way to getting the answer. Remember what we did with 5 + 6 ? Add 4 to 6 to make 10, then take 4 away from 5 to get

> 1. That gives 11. Now that we have 25 + 16, you add the 10 from the 11 with the 2 and 1 tens to make 40." The teacher writes "Answer: 41 bags" on the chalkboard.

In a brief explanation during a segment devoted to questions-and-answers and seatwork in the same lesson, the teacher mostly gave the students some problems, told them to write the equation, and occasionally gave an explanation. Here is a portion of that segment:

> Teacher writes problem on the chalkboard: "Yuichi has 14 flatfish at home. If they eat five of them today and another three tomorrow, how many will be left?" Students have different equations and teacher calls on two students to write their equations on the chalkboard. One student wrote 14 – 5 – 3 and another wrote 14 – 8. After eliciting the students' explanations for producing these different equations, the teacher explained "What's eaten is eaten. Makes no difference whether you ate it yesterday or today. They all vanish into your stomach." and then explains that 14 – 8 is the simplest way of writing the equation.

Although the majority of the segment was devoted to students writing equations, the teacher also provided explanations about the students' equations, which were quite similar to the explanations she had provided in the extended explanation segment.

Frequency of Explanations

Analyses were conducted first on the extended explanations. The Japanese children participated in significantly more lessons that contained an extended explanation than the children in the other countries, $F(2, 77) = 41.99, p < .001$. Only 1 of these was found in the observations in Taiwan and only 22 in the observations in the United States (found in 18 of the observed lessons), compared to 73 (found in 47 out of the 80 observed lessons) in Japan. Although the distribution of extended explanations was rather skewed, the analyses performed (i.e., analyses of variance) are robust with regard to skew. Post hoc Scheffé comparisons indicated that the Japanese children participated in significantly more lessons with extended explanations than either the Chinese children, $F = 32.84, p < .05$, or the U.S. children, $F = 25.96$, $p < .05$, who were not significantly different from each other.

Because extended explanations were so rare in the Chinese and U.S. observations, analyses were also conducted on explanations that lasted less than 1 min, or "brief" explanations. Significant differences across countries were found in the number of brief explanations to which children were exposed, $F(2, 77) = 7.74, p <$

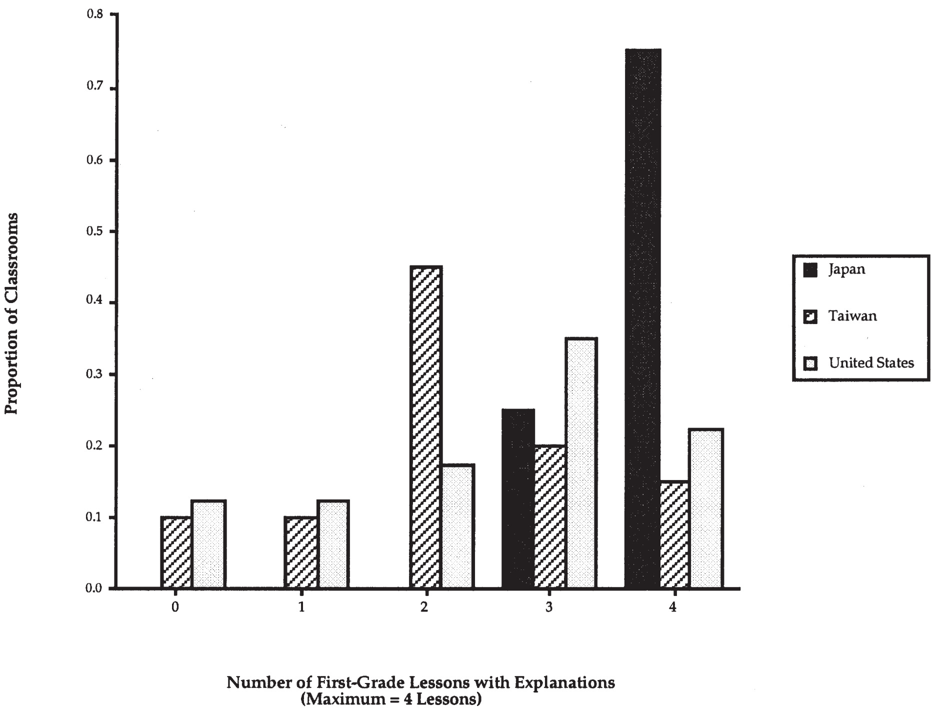

FIGURE 1 Proportion of classrooms, within country, in which explanations were observed in one, two, three, or all four of the lessons in the first grade.

.001. On average, Japanese children were exposed to 2.18 brief explanations per lesson, Chinese children were exposed to 1.76 brief explanations per lesson, and U.S. children were exposed to 1.02 brief explanations per lesson. The post hoc comparisons showed that the Japanese lessons contained more brief explanations than the U.S. lessons, $F = 6.97, p < .05$.

Besides differences across countries, there was also variation across classrooms, within countries, in terms of the frequency with which explanations were provided. In general, most Japanese children were exposed to at least one explanation (brief or extended) in three or all four of the observed lessons. Thus, there was no significant variation across the Japanese classes because most classes received explanations in the majority of the lessons that were observed ($SD = 0.44$). Significantly more variation was found across the Chinese classes ($SD = 1.15$) than in the Japanese classes, $F = 9.56, p < .05$, and significantly more variation was found in the U.S. classes ($SD = 1.32$) than in the Japanese classes, $F = 9.32, p < .05$. The data supporting these claims are presented in Figure 1. Although there was considerable variation in the number of explanations provided across the U.S. classes that were observed, this variation was not systematic, $F(2, 39) = 0.06$, across types or location of school (i.e., Chicago public schools, suburban public schools, and private schools) in contrast to presumed superiority of the suburban public schools and private schools over the Chicago public schools (e.g., Bryk, 1993).

Duration of Extended Explanations

Durations of segments coded as extended explanations were analyzed across sites. Average durations of these segments did not differ significantly. The mean durations of the segments with extended explanations ranged from 5.03 min in the United States to 5.5 min in Taiwan to 5.91 min in Japan. Although the average durations were not significantly different, given that the Japanese children received more explanations than children in the other sites, it is clear that the Japanese first-grade children spent more total time devoted to explanations than the other first-grade children in the sample.

Types of Explanations

Although Japanese children were spending more time in lessons that contained explanations, it remains unclear whether these explanations were qualitatively different (and potentially better) than explanations that children were exposed to in the other sites. To begin to address this issue, analyses were performed on which mathematical topics were explained, and then further analyses were performed on explanations occurring within frequently occurring topics. Separate analyses were conducted for brief and for extended explanations; however, it is important to keep in mind that the sample of extended mathematical explanations in Taiwan and the United States was very small, thus permitting only limited comparisons.

Analysis of topics receiving explanations. It was important to know how the explanations were distributed over topics simply because it was not expected that explanations would be comparable across topics. Thus, types of explanations were first categorized by the mathematical topic they were intended to elucidate, and then the percentages of kinds of explanations per topic were computed.

Table 2 presents the number of explanation segments and the number of explanations occurring in nonexplanation segments in the observed first-grade lessons (i.e., extended and brief explanations). As Table 2 indicates, most explanations were about addition and subtraction, across all three sites. However, the Japanese children heard explanations about multidigit addition and subtraction (50% of all explanations were in the service of this topic), whereas the Chinese children heard

TABLE 2
Topics Receiving Explanations in First-Grade Classrooms

	Number of Extended Explanations			*Number of Brief Explanations*		
Topic	*Japan*	*Taiwan*	*United States*	*Japan*	*Taiwan*	*United States*
Single-digit addition and subtraction	1	0	7	4	72	39
Multidigit addition and subtraction	31	0	6	100	5	17
Place value	28	0	3	41	1	10
Measurement	0	1	3	0	37	8
Calendar	0	0	1	0	30	5
Geometry	13	0	0	29	0	2
Fractions	0	0	1	0	0	33
Time	0	0	1	0	0	27
Money	0	0	0	0	1	12
Graphs	0	0	0	0	0	6
Other	0	0	0	2	0	1
Total	73	1	22	176	146	160

explanations about single-digit addition and subtraction (52% of all explanations), and the U.S. children heard about both (23% about single-digit and 12% about multidigit addition and subtraction). Thus, by the second semester of the first-grade year, the Japanese students were consistently receiving explanations concerning multidigit addition and subtraction problems (as well as the related topic of place value). The Chinese students were consistently receiving explanations concerning more simple addition and subtraction problems (with a lesser but not insignificant focus on measurement and calendar problems). With respect to addition and subtraction concepts, the U.S. students were receiving explanations on both single-digit and multidigit addition and subtraction problems; keep in mind, however, that U.S. students were also hearing explanations about a wider variety of topics than their Asian peers. This greater variability in terms of the types of topics receiving explanations in the United States probably reflects both the diversity within the U.S. sample and the stricter control of curriculum in the two Asian sites.

An issue that is raised by these data is whether the Japanese children look superior to their Chinese and U.S. peers on achievement tests simply because the Japanese teachers are moving at a faster pace relative to the teachers in the other two sites. This is directly related to the preponderance of explanations because it is easy to imagine that the more topics that are introduced, the more explanations are called for. To explore this issue, the number of topics presented in the 2-week observation period was analyzed. In fact, the Japanese classrooms appeared to be moving at a slower pace than classrooms in the other two sites and covered significantly fewer topics during the observations than the Chinese or U.S. classrooms:

The average number of topics in the Japanese classrooms was 2.35 ($SD = 0.99$), the average in the Chinese classrooms was 3.55 ($SD = 1.0$), and the average in the U.S. classrooms was 4.17 ($SD = 2.01$), $F(2, 72) = 8.72$, $p = .0004$.[2]

Analysis of addition and subtraction explanations. It was also important to consider whether the explanations were communicating the same depth of information. Thus, to look at whether the explanations across the three sites were comparable in this regard, addition and subtraction (combined for single digit and multidigit) were chosen for close examination. Explanations about addition and subtraction were chosen for two reasons. First, as was discussed earlier, it was important to limit comparisons to one topic because it was possible that certain topics could elicit more complex or more systematic explanations than other topics. Second, there were more of these kinds of explanations than for any other topic (49% of all the explanations across the three countries were concerned with these topics), and they were distributed fairly evenly across countries, suggesting that this was an important topic to explain to first-grade children, in general.

The explanations of simple addition and subtraction problems were combined with explanations of multidigit problems because the sorts of explanations used across the simpler and more difficult problems were comparable. For example, it was quite common for teachers to provide an explanation using an example problem for a single-digit or a multidigit problem. There are two notable exceptions to the use of the same explanatory strategy across problem types. First, explanations relying on place-value concepts typically occurred for multidigit problems and not for single-digit problems because the issue of place value is not particularly relevant to single-digit numbers, especially when compared to multidigit numbers. Second, explanations relying on a family metaphor typically were provided for single-digit problems and not for multidigit problems. Families were described either as groups of related numbers or related facts. Sometimes teachers explained that a set of numbers, such as 4, 5, and 9, belonged together as a family (because, for example, $4 + 5 = 9$ and $9 - 5 = 4$); other times teachers explained that a set of facts were related (e.g., both $2 + 9 = 11$ and $4 + 7 = 11$ belong to the "11 family"). Ostensibly, the purpose of providing these sorts of explanations is to demonstrate commutativity and to facilitate computation, although these purposes were rarely explicitly stated to students.

The listing of the types of addition and subtraction explanations, with examples, can be found in Table 3. Cohen's kappa was .81 for identifying specific types of explanations on 50 randomly selected explanations. It should be noted that almost all of the explanations were explanations of procedures for solving addition and subtraction problems, although a few highlighted arithmetic or mathematical principles. Conceptual explanations and explanations of principles were rare, and

[2] I thank an anonymous reviewer for suggesting that I explore this point.

TABLE 3
Explanations of Single- and Multidigit Addition and Subtraction Problems by First-Grade Teachers

		Percentage of Types Used in		
Type	*Example*	*Japan*	*Taiwan*	*United States*
Explanations that Generalize Beyond the Specific Problem				
De- and recomposing to 10	"I have 8 and get 5 more, so I add 2 to 8 and makes it to 10 and then add the 3 left there."	3	78	6
Alternate solution methods and principles of addition	"It doesn't matter whether one starts with 30 or 20 first when adding 20 + 30."	17	8	9
Place value	"2 tens plus 3 tens … 2 tens is 20 and 3 tens is 30."	51	0	21
When to use which operation	The teacher explains that problems with joining or the clue word *together* hints that it is an addition problem.	13	3	0
Tricky problems	One pencil costs $8. How much for 2? Would adding 2 magnets to 8 show the problem? Explanation: This problem requires adding 8 to 8.	0	6	0
		84[a]	95[a]	36[a]
Specific Explanations				
Example problem	The teacher says "Explain the equation 46 + 21 = ?" The student explains: "Add 6 to 1 to get 7, then add 4 to 2 to get 6."	10	0	17
"Subtract the smaller from the larger number"	The teacher asks: Why subtract 32 from 46? A student explains: "subtract the smaller number from the larger one."	0	0	9
Count to add	To add 1 + 2, put one pea and then 2 more peas, then count the total.	0	0	17
Families	12 – 5 must be 7 because they are a family of numbers.	0	0	15
		10[b]	0[b]	58[b]
Other		6	6	6
Number of explanations		107	36	47

[a]Proportion of generalizable explanations. [b]Proportion of specific explanations.

this was true for each of the observation locations. Even among the explanations of procedures, some explanations provided information that was more useful at the global level (i.e., for operative concepts more generally) and others provided information that was more useful at the local level (i.e., at the level of the particular problem). Explanations were coded with this distinction in mind. In the sense that more global, more generalizable explanations are potentially more powerful and sensible ways of dealing with addition and subtraction concepts, it appears that the Asian children in this sample were exposed to relatively more powerful and sensible ways of dealing with addition and subtraction concepts than their U.S. peers.

Descriptive analyses of first-grade explanations. Some examples of the types of explanations serve to illustrate differences in the quality of mathematical explanations and the notable distinctions across countries. To begin, almost all (78%) of the Chinese explanations depended on decomposing and recomposing numbers to 10. For example, one teacher explained that in adding 7 + 8, you can break the 8 into 3 + 5, add the 3 to the 7 to get 10, then add the 5 to the 10 to get 15. The value of this sort of explanation lies in its reliance on a base-10 rationale and, thus, on its generalizability to addition and subtraction of larger numbers. Although the explanations typically laid out the procedure for decomposing one addend to be recomposed with the other addend to form 10, these procedural explanations were often accompanied by functional explanations (e.g., we do this to make addition simpler or when adding 5 + 8, moving 2 from the 5 to be combined with the 8 is faster than moving 5, when using manipulatives, so the first method of moving only 2 is better than the second method). In this way, these students were in a position to understand not only how to use the procedure but also the conditions under which the procedure should be used. More than half (51%) of the explanations of addition and subtraction in the Japanese data were place-value explanations. An example of this type of explanation is, "When adding 63 and 26, you can break the 63 into 60 and 3, and you can break the 26 into 20 and 6. Because you know what 3 and 6 is, and because you know what 60 and 20 is, you can easily solve the problem." These sorts of explanations potentially allowed children both to understand composition of numbers (e.g., that 63 is 60 and 3 more) and to understand that additive principles that apply for one place value also apply for other place values. It is likely that the composite and consistent structures of numbers were made apparent by relying on place value to explain addition.

In general, the U.S. children heard types of explanations that the Asian children did not. The predominant types of explanations that U.S. children heard included place value, example problems, counting to add, and families. Each of these deserves a further description. The place-value explanations that U.S. children heard were comparable to those just described for the Japanese data. The example problems were also similar to those identified in the other locations. In general, these explanations consisted of teachers explaining the procedure for how to solve a particular problem. Of the types of explanations that were common in the U.S. data, two were unique to U.S. classrooms: counting to add and families. The counting-to-add explanations simply informed the children that one way to find the solution was to count (count all or count up). Finally, U.S. children learned about families of numbers, or numbers that go together, and their Asian peers did not. It is also worth noting that the explanations based on the families metaphor could be difficult to understand (at least by the adult coders) or were often too literal to be used sensibly. As an extreme example, one first-grade teacher explained families in the following way: "When you're working with a family of numbers, you only have those three numbers … the family consists of a daddy, a mommy, and a baby." At worst, it is easy to imagine much misinterpretation of addition from this metaphor; even at its best, this explanation provides no real mathematical meaning. Although other teachers were much better than the one represented here at using this metaphor to explain relations among numbers, as this example illustrates, the metaphor of families is not straightforward and may not always be useful.

What we learn from these examples is that the Chinese and Japanese children most often were presented with what seemed to be consistent, straightforward, and useful methods for solving addition and subtraction problems. In comparison, the U.S. children were often presented with ways of handling these same problems that were either difficult to understand (e.g., the families explanation) or difficult to generalize to more difficult problems (e.g., it would be difficult to count on your fingers to solve multiplication problems or even to subtract 37 from 82).

One other related difference of emphasis is worth noting. In the United States, but not in the Asian countries, children heard admonitions about never subtracting the larger number from the smaller number. Although this may have made sense for the problems that the children were given in the first grade, eventually this will turn out to be bad advice and may, in its own way, discourage children from thinking that there may be such things as negative numbers (also see Ma, 1999, p. 3).

In sum, for the first-grade classrooms, explanations were more frequent in the Japanese classrooms than in the classrooms observed in the other sites. Moreover, typical explanations in the Asian classrooms were more substantive than those observed in the U.S. lessons. Together, this suggests that Asian children were advantaged relative to U.S. first-grade children in terms of having access to useful mathematical explanations.

FIFTH GRADE

The next set of analyses were devoted to investigating whether explanations were different across the three sites in the fifth grade. Three specific questions were addressed. First, what is the general context of the observations? Second, how frequently were fifth-grade children engaged in discourse focusing on mathematical explanations? Third, what was the character of these explanations?

Description of Classroom Activities as Contexts for Explanations

The most commonly occurring activities in the fifth grade, as in the first grade, were question-and-answer periods, seatwork, or question-and-answer periods intermingled with seatwork. The frequency of activities and proportion of these activities that contained explanations are displayed in Table 4. Across all three observation sites, these accounted for 58.9% of all 2,132 segments (60.4% of all segments in Japan, 55% in Taiwan, and 60.4% in the United States). As in the first grade, many of these types of segments (39%) included explanations: 39.5% of the Japanese, 36.4% of the Chinese, and 40.4% of the U.S. question-and-answer and seatwork segments contained an explanation. Explanations were also found in evaluation segments ($n = 134$, or 40.8% of the evaluation segments, which is a substantial percentage of all evaluation segments but accounts for only 6.3% of all segments in the corpus). Of course, explanations appeared in activities that were classified as explanation activities. A substantial proportion (34%) of all explanations occurred in the context of activities that could be characterized as extended explanations. Explanations rarely appeared in the other types of activities. Unlike the data from the first-grade observations, the focus of the remaining analyses are on extended rather than on brief explanations. This decision was based primarily on two rationales: Extended explanations were common (especially compared to what was found in the first grade), and a substantial proportion of segments containing explanations were, indeed, explanation segments.

TABLE 4
Activity Frequencies in Fifth-Grade Classrooms

	Occurrence (n)			*Percentage of Activity That Included Explanation*		
Activity	*Japan*	*Taiwan*	*United States*	*Japan*	*Taiwan*	*United States*
Question-and-answer	213	207	282	58	52	59
Seatwork (SW)	152	113	214	13	8	14
Question-and-answer with SW	25	10	39	44	30	49
Explanation	124	108	97	100	100	100
Evaluation	108	105	115	48	45	30
Choral responses	1	12	0	0	8	—
Teacher gives directions	4	3	47	25	67	13
Mental calculation	1	25	0	0	16	—
Other	18	17	92	11	0	3
Total	646	600	886	52	47	40

Frequency of Explanations

The average number of lessons, out of the four lessons observed, which contained an extended mathematical explanation, were compared. An analysis of variance revealed significant differences in the number of extended explanations that children heard in the three countries, $F(2, 77) = 16.57, p < .001$; the mean number of explanations in Japanese lessons was 3.35 (out of the 4 observed lessons), 2.9 in Taiwan, and 1.5 in the United States. Post hoc comparisons showed that the Japanese and the Chinese lessons contained significantly more explanations than the U.S. lessons (using Scheffé comparisons, $F = 13.48$, comparing Japan and the United States, and $F = 8.30$, comparing Taiwan and the United States, $p < .05$ for both). The Japanese and the Chinese lessons did not differ significantly from one another, $F = 0.47$.

Besides differences across countries, there was also variation among classrooms, within countries, in terms of the frequency with which explanations were provided (see Figure 2). As in the first grade, most Japanese children were exposed to an explanation in three or all four of the observed lessons. More variation was found among the Chinese classes than in the Japanese classes, but more than half of the Chinese classrooms received explanations in all four of the observed lessons. The greatest amount of variation was found in the U.S. classes; less than 40% of the classrooms had explanations in more than two of the four observed lessons. Although there was considerable variation among U.S. classes, this variation was not systematic: No significant differences were found across type of school (i.e., Chicago public schools, suburban public schools, and private schools), $F(2,39) = 0.80$.

Duration of Explanations

The average durations of extended explanations were not significantly different across the three sites, with average durations of extended explanations ranging from 3.76 min in the United States, to 4.29 min in Taiwan, to 4.61 min in Japan. Although the average durations were not significantly different across sites, when the average durations are multiplied by the frequency of explanations, it is apparent that the Asian children in this sample spent more total time devoted to mathematical explanations than U.S. children.

Types of Explanations

Analysis of topics receiving explanations. To investigate the types of explanations children heard, it was important to know how the explanations were distributed over topics because it was not expected that explanations would be compa-

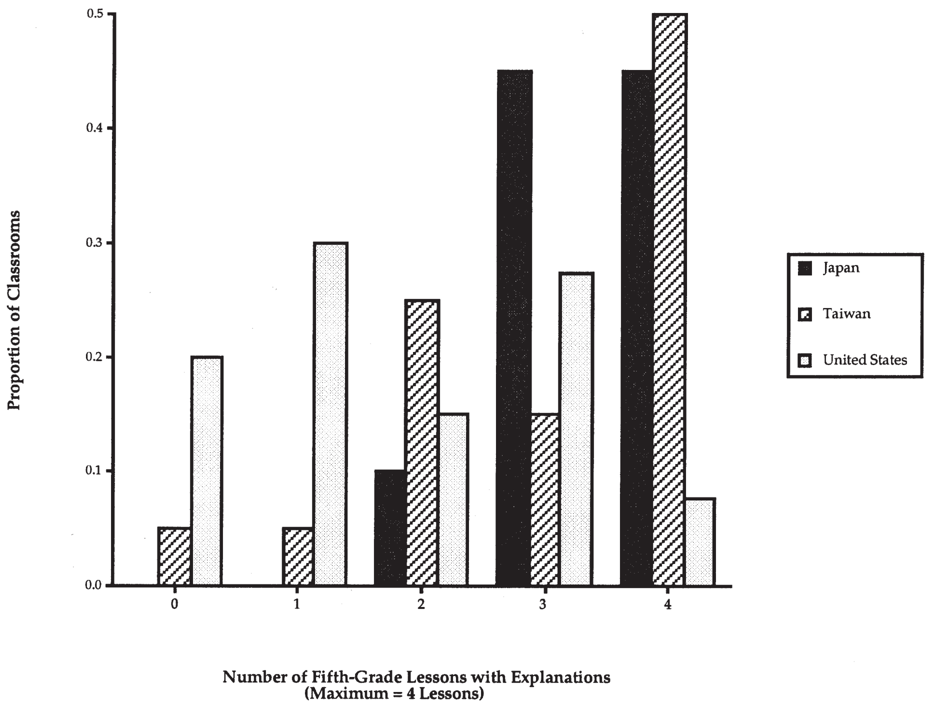

FIGURE 2 Proportion of classrooms, within country, in which explanations were observed in one, two, three, or all four of the lessons in the fifth grade.

rable across topics. As with the first-grade data, the types of explanations were categorized by mathematical topic, as reported in Table 5.

The data in Table 5 indicate that over 50% of all explanations in each of the three sites were about fractions (including fractional concepts, such as ratios and proportions). This is not to say that all problems about the same topic were equivalent in terms of difficulty but merely to say that, across the three sites, children were working on a set of concepts that might evoke comparable sorts of explanations. The Chinese and the U.S. children also heard many explanations about measurement (21.3% and 15.5%, of all explanations heard by children in Taiwan and the United States, respectively), and the Japanese heard many explanations about graphs (35.5% of all explanations) and averages (17.7% of all explanations).

To understand whether the Asian children were hearing explanations that were somehow more sophisticated or more powerful than those heard by the U.S. children, it was important to control for what topic was being explained. Because so many of the explanations heard in each of the three sites concerned fractions and related concepts (51.4% of the 329 extended explanations recorded in the data) and

TABLE 5
Topics Receiving Explanations in Fifth-Grade Classrooms

	Number of Segments		
Topic	*Japan*	*Taiwan*	*United States*
Fractions	55	58	56
Measurement	0	23	15
Graphs	44	0	1
Time computation	0	26	0
Averages	22	0	0
Number theory	0	0	10
Multiplication and division	0	0	7
Other	0	1	8
Total	124	108	97

because the proportion of explanations about fractions were comparable (i.e., about 50%, ranging from 44% to 58%, in all three sites), fractions were chosen for careful examination.

Analysis of fraction explanations. Teachers used four major types of explanations to communicate information about fractions: (a) explaining alternate solution methods, (b) explaining the relations among component parts of a problem (including their definitions), (c) working through an example problem, or (d) providing a rule or directive. Table 6 provides the proportion of each type of explanation used in Japan, Taiwan, and the United States, along with an example of each type. Cohen's kappa was .81 for identifying specific types of explanations between two coders (on 50 randomly selected explanation segments).

As can be seen in Table 6, the use of these four types of explanations differed dramatically across the three sites. Most of the Japanese explanations reminded children that fractions are composed of component parts and that the relation between these parts is important to consider. The explanations provided in the Chinese classrooms were more eclectic: Concepts were explained by pointing out alternate solutions, describing component parts, and providing examples. In the United States, it was rarely pointed out that there might be more than one way to solve a problem; instead many explanations consisted of rules or directives that lacked a description of why the rules were important. For example, many of the U.S. explanations could be characterized by the example, "Always reduce to lowest terms." This explanation does not tell the children how or why to reduce to lowest terms but counts as an explanation in the sense that the children were hearing about why they were to do something in the sense that we always do this.

DISCUSSION AND CONCLUSIONS

Three major results deserve further examination and discussion. First, the Asian children in this sample received more explanations than the U.S. children. Second, in the first grade, the Japanese children were learning about more complex topics than their peers in Taiwan or in the United States. Third, the quality of the explanations appeared to differ across the three sites. The question that must be asked of each of these results is, how might this affect children's abilities to develop mathematical knowledge?

The Amount of Explanations

In general, the Japanese and Chinese children heard more explanations than their U.S. peers. What is the impact of hearing more explanations (independent of the issue of hearing potentially better explanations, which is discussed later)? Two advantages seem apparent. First, if children hear explanations often, they are more likely to develop a notion that explanations are an appropriate form of discourse in mathematics classes than if they only hear explanations rarely. In other words, when children are in environments where explanations are common, they are more likely to adopt this discursive form than when explanations are rare. This parallels the finding reported by Moely et al. (1992) in that students who are exposed to more explanations have a better basis for making sense of explanations of novel content because they presumably have already discovered that explanations are useful (also see Fernandez, Yoshida, & Stigler, 1992).

TABLE 6
Explanations of Proportions and Fractions by Fifth-Grade Teachers

Type	Example	*Percentage of Types of Explanations Used*		
		Japan	*Taiwan*	*United States*
Alternate solution methods	"You can either multiply by 1/3 or divide by 3. The result is the same."	16	35	3
Component parts of a problem (including definitions and relations)	"Percent is dependent both upon the amount compared and the total amount."	55	28	32
Example problem	"In 3/8 + 1/8, add the numerators, which is 4, and leave the denominator as 8."	25	28	37
Rule or directive	"Always reduce to lowest terms."	4	9	29
N of explanations		55	58	56

Second, receiving many explanations makes it more likely that children eventually will come to understand a concept than if they receive explanations infrequently. In other words, explanations typically contain useful information, so the more explanations children hear, the more likely it is that they will be exposed to useful mathematical information. In this sense, the Asian children in this sample, especially the Japanese children, appear to be at an advantage because they were engaged in considering more explanations than their U.S. peers.

Alongside the benefits of simply hearing more explanations lies a potential cumulative effect. Although data were collected only for first and fifth grades, the trends in the data suggest that mathematical explanations are a part of the fabric of Japanese and Chinese lessons and more like loose threads in U.S. lessons. Given this scenario, the Asian children in this sample were placed in an increasingly good position to make use of the explanations: If they hear explanations often, students have probably developed ways to understand how to use them. For U.S. children, explanations were relatively rare and may be presented inconsistently throughout the elementary years, potentially risking the possibility that they would be ignored or perhaps difficult to use appropriately. In other words, the teachers were not practiced at giving explanations, and the students were not practiced at incorporating explanations, relative to the teachers and students in the Asian classrooms that were observed.

The Complexity of the Topic Explained

Not only do the Japanese children hear more mathematical explanations than U.S. children, they also hear explanations about more complex mathematics. In the first grade, by the time the observations began, the Japanese children were already focusing on multidigit addition and subtraction. Because simpler concepts are often prerequisites for understanding difficult concepts (e.g., multidigit addition requires the ability to solve single-digit addition problems), it is possible that the first-grade Japanese children were developing more sophisticated mathematical knowledge than their peers in Taiwan or in the United States. We can only speculate about the influence of the early introduction of complicated mathematical concepts. However, previous work has shown that covering even very few difficult mathematical concepts puts children at an advantage on tests that include those concepts; likewise, if those concepts are not covered, children can have difficulty solving problems that are based on those concepts (e.g., Perry, 1988).

It is worth pointing out that although the observed Taiwanese first-grade lessons apparently were less complex than the observed Japanese first-grade lessons, in the long run this does not disadvantage the Taiwanese students: By the fifth grade, these students perform significantly better than the U.S. students but not significantly differently from the Japanese students. Why is this initial lack of complexity, indeed a focus on simplicity, not a problem? Ma (1999) wrote that, in her investigation of U.S. and Chinese mathematics teachers, "most of the Chinese teachers mentioned the issue of 'subtraction within 20' as the conceptual, as well as procedural, 'foundation' for subtraction with regrouping" (p. 15). The teachers identified by Ma as having profound mathematical understanding purposefully spent much time making sure that their students understood the elementary mathematics that is the basis of all mathematics. To confirm this point, Ma commented that:

> The Chinese teachers believe that if students learn a concept thoroughly the first time it is introduced, one will get twice the result with half the effort in later learning. Otherwise, one will get half the result with twice the effort. (p. 115)

It may be that by spending so much time on the basics of mathematics, the Chinese students ultimately are advantaged in understanding more complex mathematics.

Note that just because the Japanese first-grade children heard explanations about relatively more complex mathematics than the other first-grade children does not mean that the Japanese proceeded through the curriculum at a fast pace. Indeed, as others (e.g., Stevenson & Stigler, 1992; Stigler et al., 1996) have reported, the Japanese classroom appears to move at a slow pace. As Stevenson and Stigler wrote: "American elementary school students, watching a videotape of a Japanese mathematics lesson, inevitably react to the pace: They perceive an unbearable slowness" (p. 194). This slower pace, along with more explanations, may work together to assure that Japanese children understand the mathematics introduced in the curriculum. Indeed, rapid switching of topics was documented here for the Chinese and U.S. classrooms relative to the Japanese classrooms. The rapid topic switching, which may indicate rapid movement through the curriculum, may add to the confusion and lack of coherence, thereby interfering with the development of mathematical knowledge (e.g., Stigler et al., 1996; Stigler & Perry, 1988b).

The fact that Japanese children were exposed to fewer topics raises the issue that the consistency of the curriculum (both within and across classrooms) might affect explanations and learning outcomes. More specifically we can ask how a national curriculum might impact mathematics teaching and learning. The two sites that adhere more or less (Taiwan more and Japan less) to a national curriculum have students who perform better, as well as teachers who explain better, than the one site (United States) in which curriculum varies considerably across schools and teachers within schools. The important question here is, can the consistency across classrooms influence the explanations that were observed? It is likely that the consistency at least contributes to these observed differences across sites. For example, it is easy to imagine that if the teachers, across classrooms, were working with the same materials, they could communicate with each other more easily and thus could collaborate on crafting useful explanations than when teachers use different curricula (for a similar argument see Stigler & Hiebert, 1999). Moreover, given the relatively high transience of U.S. schoolchildren—which implies changing teachers, classmates, neighborhoods, dwellings, and the like—we could make things easier on them by not also having to change curricula when they move. The observational data raise this question but cannot answer it, so these issues must be pursued and examined more carefully in future research.

Quality of Explanations

One premise of this investigation was that we cannot fully understand the cross-national differences in achievement without examining what happens in the classroom. Also, it is clear that this issue cannot be addressed satisfactorily without some attention to the quality of the explanations that were observed in the classrooms.

Although the issue of quality has been skirted thus far, primarily because of the difficulty of assessing goodness, some initial evaluations can be made. First, as already discussed in the descriptive analyses of the explanations, the explanations provided in the Japanese and Chinese classrooms apparently were more generalizable across problems than the explanations in the U.S. classrooms, and thus these explanations potentially are more powerful than the explanations that are more useful for individual problems. Second, students in each of the sites heard explanations about how to solve problems, but not all students heard explanations of mathematical principles and functions (i.e., explaining the function of a particular mathematical procedure or concept). Arguably, if a student can know why a procedure works and when to use it, that student will be better equipped to handle novel problems and use these learned procedures than a student who does not know these things. Thus, the Asian children in this sample were advantaged again because they received principle and functional explanations, which potentially are more useful and better than rarely or never providing these sorts of explanations.

It is likely that at least part of the reason the Chinese and Japanese students heard better explanations relative to their U.S. peers rests on teachers' understanding of the mathematical concepts that were taught. In other words, teachers who understand the mathematics they are teaching should be better at explaining those mathematical concepts than teachers who do not have a deep and connected understanding of mathematics. Ma (1999), in her investigation of U.S. and Chinese teachers' understanding of arithmetic concepts, found that the quality of teachers' explanations were directly related to their understanding of mathematics. She found that many of the Chinese teachers, but none of the U.S. teachers in her sample, had a profound understanding of fundamental mathematics. As she put it, "from a teacher who cannot provide a mathematical explanation of algorithms for subtraction with regrouping, … what kind of 'teaching for understanding' can we expect?" (p. 152).

General Conclusions

The kind of research presented here is well suited to highlight differences in educational practices, and these differences provide implications for understanding learning processes. In particular, because Asian elementary school students outperform U.S. students on cross-national tests of mathematical understanding, it is tempting to conclude that the practices observed in Asian classrooms provide important opportunities for Asian students to construct sophisticated mathematical knowledge, relative to their U.S. peers. This does not necessarily mean that we should argue to import Asian methods into U.S. classrooms because, for example, U.S. teachers have things to contend with (including a relatively diverse surrounding culture) that the Japanese and Chinese teachers observed in this investigation do not.

All hedges aside, however, it does not seem unreasonable to advise U.S. teachers, in general, to engage in mathematical dialogue with their students and work to improve the sorts of mathematical explanations that get presented and discussed in their classrooms. Both the quantity and quality of mathematical explanations observed in Asian classrooms potentially lead children to form rich mathematical knowledge. For example, without hearing how multidigit addition and subtraction depends on place-value concepts, it is likely that this critical component of understanding addition and subtraction could be overlooked and not integrated into addition and subtraction concepts. Children's development of buggy subtraction algorithms, like those reported by Brown and Burton (1978) and VanLehn (1990), seem inevitable without a firm understanding of place value, especially if children also learn that they should be subtracting smaller from larger numbers. Thus, to achieve successful mathematical understanding, we must go beyond telling children how to solve mathematical problems; we must reach a point where children are not only successfully producing mathematical solutions but also understanding why the procedures work and when the procedures are and are not applicable. This point may be reached by providing children with, and requiring that they contribute, to adequate explanations in their mathematics classrooms.

In conclusion, the ways in which mathematical concepts are explained to children may contribute to the well-documented differences in mathematical performance across countries. The data provided by observations of classrooms can provide insight into how mathematical concepts are shaped for children in different learning contexts. This insight may help us to begin to understand why U.S. children achieve so poorly in mathematics compared to their Asian peers.

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