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Learning Mathematics in First-Grade Classrooms: On Whose Authority?

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Integral to knowing mathematics is an understanding of how mathematical ideas are generated and validated. In mathematics classrooms, teachers socialize this understanding by establishing discourse patterns and participatory structures in which various sources—the teacher, the text, the discipline of mathematics, or the community of learners—are implicitly and explicitly credited with the authority to develop and validate mathematical ideas. In the present investigation, the authors focused on classroom discourse processes and participatory structures that grant sources of mathematical authority in 6 first-grade classrooms. In general, teachers firmly and with few exceptions positioned themselves as the sole mathematical authority in their classrooms. Yet, the authors found significant exceptions in 1 teacher's lessons, with these exceptions inspiring possibilities in accomplishing the shift from a formal to a growth-and-change tradition of socializing students into the discipline of mathematics.

Mathematical ideas, although oftentimes presented in textbooks, originate from human experience. Still, many of us turn to texts and to human experts to verify our ideas. Although there is nothing wrong with checking our ideas and formulations against those presented in books or by experts, this practice potentially hides the fact that these ideas and formulations originated from people.

The fact that mathematics is a human invention (albeit, invented by extraordinary humans) may seem obvious, but it is not obvious to young students in American schools. Most students act as if mathematical ideas are predetermined and unarguable truths, which they either do or do not understand, but not ideas to which they could possibly contribute to or question (Schoenfeld, 1992; Stodolsky, 1988). Why would this be the case? In other words, why would students act as if they have no say in either the creation or verification of mathematical ideas? And, if we are to take recent reforms in mathematics education seriously, to have students vitally involved in conducting mathematical inquiry, how can we change this and promote students to take an active role in the creation and verification of mathematical ideas? These questions guide the study presented here.

A Case for Student Mathematical Authority

Clearly, in first grade, students are not creating new mathematical knowledge; a primary task is to master factual, conceptual, and procedural knowledge that is known and accepted. Commonly used traditional instructional practices such as teacher-guided, procedure-based instruction and student drill-and-practice are typically adequate to bring about procedural fluency with respect to basic algorithms (Hiebert & LeFevre, 1986). But to participate in mathematical activity, students need facility with complementary processes of mathematical thinking: deductivism and empiricism (Schoenfeld, 1986). That is, students must become proficient with what is already known by mathematicians (e.g., syntactic rules, algorithms, symbols) and also develop the ability to apply a formal understanding of mathematics to generate and to provide support for new mathematical ideas. In contrast to procedure-based, teacher-led instruction, pedagogy that grants learners the authority to connect prior conceptions to novel information and to generate, apply, and verify solutions to problems provides for greater conceptual understanding and transfer (Bransford, Brown, & Cocking, 1999; Hiebert & Wearne, 1992; Perry, 1991). Moreover, at this early point in their mathematical careers, first-grade students learn more than facts, procedures, and concepts: They learn important dispositions toward mathematics as a discipline that incline their mathematical engagement and understanding (Lampert, 1990). A strong emphasis on the scripted construction of mathematical ideas and procedures without attention to empirically based idea generation and verification by students may foster beliefs that the primary objective in mathematics is to reach a correct answer (Schoenfeld, 1986), which may ultimately lead students to disengage from mathematics as a discipline (Boaler & Greeno, 2000).

Sources of Mathematical Authority

Lakatos (1976) argued that, within the discipline of mathematics, perspectives on how mathematical ideas and truths are devel-

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oped and validated can be described as belonging to a "growth-and-change" view (Lampert, 1990). This view emphasizes that mathematical ideas and truths are developed, granted significance, and verified through mathematical reasoning and acceptance by an intellectual community. As Ma (1999) noted: "Arithmetic, as an intellectual field, was created and cultivated by human beings. Teaching and learning arithmetic, creating conditions in which young humans can rebuild this field in their minds, is the concern of elementary mathematics teachers" (p. 114). This idea stands in contrast to a formalist view in which foundations of mathematical ideas exist in terms of immutable truths and unquestionable certainty, independent of human experience (Lampert, 1990).

A growth-and-change view of mathematics is a cornerstone of recent mathematics reform of the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000; Cooney & Shealey, 1997). In particular, the NCTM has made it clear that students should be making mathematical conjectures, justifying their own reasoning, and questioning their own and other's thinking: "If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential" (NCTM, 1989, 1991, 2000, p. 18). Such orientations require that members of the intellectual community, which includes students and the teacher, have the authority to develop and validate mathematical knowledge. Correspondingly, the authors of the NCTM standards called for a deemphasis on the teacher or text as sole authority for correct answers and a concomitant emphasis on logic and mathematical evidence provided by members of the mathematics-classroom community for verification of ideas and knowledge.

Teachers socialize a sense of mathematical authority through multiple formats. Teachers can create classroom climates rich with the *analysis* of mathematical ideas through mathematical *discourse* or through a mutual exchange of ideas that are debated and defended by students and the teacher (NCTM, 2000). Routine engagement in these two processes contributes to students' development of mathematical authority in two critical ways. First, students gain first-hand experience with respect to the behaviors of members of a mathematical community by using mathematical terminology, making mathematical conjectures, and arguing for and against mathematical ideas. Second, high levels of mathematical discourse and mathematical analysis communicate an implicit message that mathematics is a discipline in which ideas are negotiated and verified through mathematical evidence, by a community of learners (Lampert, 1990). Furthermore, being granted the authority to propose ideas, evaluate ones' own and others' thinking, and to develop mathematical reasoning skills contributes to students' conceptual and procedural understanding of mathematics (Hiebert & LeFevre, 1986; Lampert, 1990; NCTM, 2000).

Specific teacher practices such as questioning, integration of student ideas, and verification of student ideas directly inform students as to how mathematical ideas are developed and validated. Questions that ask students to name, identify, or calculate typically call for an answer that the teacher already knows and that the student simply provides; such questions send the implicit message that mathematics is a discipline in which someone—the teacher—always knows the answers and in which creative solutions are not valued. In contrast, teacher questions that call for explanations of thinking processes or for student ideas about mathematical possibilities communicate the view that mathematics is a discipline in which there are multiple pathways to understanding that can be evaluated individually for their own merit by the intellectual community (Cobb, Wood, Yackel, & McNeal, 1992; Lampert, 1990; Stigler, Fernandez, & Yoshida, 1996; Yackel & Cobb, 1996). The manner in which teachers follow through on their questioning must also be considered for its socialization role. Cobb and colleagues (Cobb et al., 1992) provided evidence that teachers may ask the class to evaluate or verify a student's response, or alternatively, never consult the students and act as the sole validator of students' answers or ideas. The point is that an understanding of mathematical authority may be found not only in the specific questions that teachers ask but also in how teachers follow through on students' responses.

Teachers also act on student ideas in ways that communicate mathematical authority. Several researchers refer to appropriation, the processes through which teachers build on student ideas to connect them to mathematical understanding in the larger discipline (Newman, Griffin, & Cole, 1989). Cobb et al. (1992) presented examples in which teachers reformulated student ideas so that lesson objectives could be advanced while the integrity of the original student idea was maintained. Other researchers have observed teachers' practice of maintaining a link between an idea and the student who generated it as the idea is discussed and evaluated by the classroom community (Ball, 1993; Lampert, 1990; Stigler et al., 1996). Yet in other instances, teachers are described as twisting student responses to conform to mathematical convention to the point that there is little resemblance to the original student idea (Cobb et al., 1992; Stigler et al., 1996). Such interpretations of student ideas provide subtle messages communicated to students about who is the ultimate source of valid ideas and verification of knowledge in mathematics.

Can There Be a Community of Mathematical Authorities?

Clearly, there is potential for communities of learners in American classrooms who assert mathematical authority even in the early elementary grades (e.g., Cobb et al., 1992; Lampert, 1990; Peterson, 1993). As an example, Peterson (1993) provided an excerpt from a classroom transcript in which the teacher and students passionately discussed whether 0 is positive, negative, or a divider between positive and negative numbers. After reading this exchange, it is easy to be convinced that even first-grade students can engage successfully in mathematical discourse of the sort that marks them as authorities who contribute to and verify mathematical ideas. But as Peterson clearly laid out, the teacher worked closely with Peterson, a world-renowned researcher in the area of mathematics teaching and learning. Other documented instances of classrooms in which students assert mathematical authority are similar in that the classroom teachers have had a close affiliation with researchers (see also Cobb et al., 1992; McClain & Cobb, 2001) or actually are the researcher (e.g., Lampert, 1990).

Although descriptions of model teachers lend optimism to unrealized possibilities for American classrooms, the extent to which these practices can be documented in typical American classrooms is unclear. Limited systematic attention to this question suggests that discourse practices in typical American classrooms remain driven by the teacher and text, which are established and reinforced as primary sources of mathematical authority, whereas students are relatively ignored as sources of authority. For instance, as part of the analyses of Third International Mathematics and Science Study, Stigler et al. (1996) noted that through practices such as questioning, opportunities to discuss mathematics, attributions to sources of authority, and use of student solutions, American teachers communicated to students that in mathematics there are definite answers that are known and verified by the teacher or the textbook but certainly not by the students. Furthermore, teachers typically expressed mathematical activity in terms of preestablished procedures and rewarded students for their success in following procedures rather than for their mathematical understanding or ideas. In an in-depth study of 2 teachers implementing reform curricula, Clarke (1997) noted that although teachers invited their students to share methods and solutions to problems, there was no other evidence that they recognized students as legitimate members of a mathematical community.

The objective of this investigation was to examine the extent to which, and ways in which, teachers promote a sense of mathematical authority in their students. We document this in ordinary urban classrooms; The teachers in our sample are living in a time of rapid change in mathematics pedagogy, but they had not formally adopted reforms in mathematics education. We focus on features of the classroom climate as well as specific teacher behaviors that have been suggested indirectly and directly to communicate beliefs about mathematical authority to students.

Method

Participants

Six first-grade teachers and their students, in five different urban elementary schools, participated in this investigation. All teachers were relatively typical, at least in that they were uncoached in providing a reform-oriented mathematics classroom and that they taught relatively average children in a major urban area. In one school, we had 2 teachers. One of the 2 teachers from the same school was a Spanish-speaking woman (Teacher A[1]) whose students were native Spanish speakers and who conducted her lessons in Spanish. The second teacher in this school was a White woman and all of her students were majority Latino (Teacher F). One teacher was African American and all of her students were African American (Teacher D). The remaining 3 teachers were White women: Two of these teachers had ethnically diverse classrooms (Teacher B and Teacher C) and 1 of these teachers had a majority Latino classroom (Teacher E). Teachers B, C, D, E, and F conducted their lessons in English. On average, teachers had 26 students in their class; Teacher F had the smallest number of students (22); Teacher D had the largest class (30 students).

The research team initially contacted principals about locating first-grade teachers in their schools who might be interested in the project and, as compensation, offered new curricular materials free of charge if the teachers agreed to participate. All teachers whose data are reported herein willingly signed informed consent forms. In addition, each student's parent or legal guardian also signed an informed consent form. Both the students' and the teachers' informed consent forms indicated that the classroom lessons would be videotaped and that the videotapes would be used for research purposes.

Conducting Classroom Observations

We had contacted the teachers in the early fall and asked them to notify us when they were planning to teach the first formal unit on place value. We chose place value because it is typically introduced in the first grade but remains a critical mathematical concept throughout the elementary years. We videotaped each teacher's mathematics lessons for 5 consecutive school days in the spring, when the teachers and students were already familiar with each other. In each classroom the video camera was positioned so that the primary focus was the teacher. During sessions of individual seat work or student-group collaboration, the camera panned across the classroom as the teacher circulated among students.

Coding Mathematical Authority Granting

We used two separate types of coding systems to measure mathematical authority granting. First, we used time-sample procedures to evaluate two features of the general classroom climate: analysis and discourse. We selected one lesson per teacher to use for reliability checks. Average Cohen's kappas across teachers was .79 for classroom analysis (simple agreement was .88) and .82 for classroom discourse (simple agreement was .93).

Second, we developed coding schemes to capture specific teacher behaviors that we hypothesized would communicate mathematical authority. In particular, we coded teacher questions, who or what was identified to verify student responses, and appropriation of student ideas. Simple agreement between two coders for the number of questions for each lesson was .91. Average Cohen's kappas across teachers was .87 (simple agreement was .96) for type of question. Interrater reliability, measured as the average of Cohen's kappas across teachers, for coding sources of verification was .85 (simple agreement was .98). We observed a low frequency of opportunities for which appropriation could be coded. Because of the low frequency, we did not calculate Cohen's kappa. Simple agreement for the appropriation categories was .86. More details about the coding categories, along with examples from the observation data, are provided in the Results.

Results

First, we report analyses that describe the general classroom climate with respect to mathematical authority. Second, we describe analyses of particular behaviors that teachers used to orchestrate mathematical authority. That is, we investigated specific instructional behaviors that directly communicated a sense of mathematical authority to students. We draw on data from classroom videotapes and audio transcripts to illustrate the patterns and relationships we report.

Level and Prevalence of Mathematical Analysis and Discourse

We analyzed the degree of mathematical analysis and the degree of mathematical discourse for each lesson with partial-interval time samples of classrooms. Time samples were composed of 5-min observe, 15-s record intervals. The total number of observe–record intervals completed for a given teacher on a given day depended on the length of the math period, which ranged from 20–75 min.[2] For each interval, the observer rated both the degree of mathematical discourse and the level of mathematical analysis that occurred. Both ratings were taken from Secada, Byrd, Peressini, and Castellon's (1993) Classroom Observation Scales.

[1] For purposes of referring to specific teachers, we alphabetized the last names of the participating teachers and then labeled the teachers A to F in descending alphabetical order.

[2] On average, Teacher B spent the most time in mathematics instruction (62 min); on average, Teacher D's classes were the next longest in duration (43 min). The other teachers spent, on average, 29 to 36 min per lesson. Although there were fluctuations across days for each teacher, lesson length across days for each teacher tended to be within 10 min of other days.

The rating of *mathematical analysis* assessed the degree to which students engaged in higher order thinking about mathematics such as searching for mathematical patterns, making mathematical conjectures, evaluating, arguing, and inventing original procedures. A rating of 5 was given when most students, for most of the time, were engaged in such mathematical analysis. A rating of 1 was given when there was an absence of mathematical analysis during the time interval. For an interval to be rated as 1, we had to witness all students involved in performing routine procedures, such as completing practice problems on worksheets or simply counting in sequence with the teacher. Midrange ratings (2–4) took into consideration the quality of participation as well as the number of students engaged. For example, a rating of 3 often indicated a moderately high level of analysis that involved several of the students.

The rating of *mathematical discourse* assessed the extent to which talking was used to learn and understand mathematics. This dimension was developed to capture exchange of mathematical ideas between teacher and students. A high rating on this dimension implies that the observed talk was not entirely scripted by the teacher and also builds on participants' ideas to promote shared understandings of mathematics. More specifically, a rating of 5 was given when sharing and developing collective understandings about mathematics involved virtually all students, was sustained across the entire time period, and dialogue was reciprocal, as opposed to dominated by the teacher. A rating of 1 was assigned when virtually no instances of mathematical discourse occurred (i.e., the students were engaged in a purely scripted, teacher-dominated discourse). As for mathematical analysis ratings, midrange ratings included assessment of discourse quality and number of students involved. A rating of 4 typically included a high level of discourse that was limited to many, but not the majority, of the class members.

To examine how the classroom climate might support students' development of mathematical authority, we asked whether there were differences among classrooms in terms of mathematical analysis and discourse. Given the low variability and restricted range across measures, we analyzed the data using the Kruskal–Wallis Test. We found significant differences across classrooms for both mathematical analysis, H (corrected for ties) $= 15.57$, $p < .01$, and for mathematical discourse, H (corrected for ties) $= 20.80$, $p < .001$. In particular, Teacher C's ratings for both mathematical analysis and mathematical discourse were higher than the ratings of all the other teachers participating in the study. The mean ratings, by teacher, are presented in Table 1.

Even with significant differences across teachers, mean ratings for mathematical analysis and mathematical discourse were low. In general, the low mean ratings for analysis indicate that students were engaged primarily in rote and routine procedures, with infrequent opportunities to make and justify mathematical conjectures or to evaluate or synthesize mathematical ideas or data. The low mean ratings for discourse indicate that classroom dialogue was primarily scripted by the teacher, with isolated and nonsustained pockets of reciprocal exchange of ideas among teachers and students.

To illustrate the range of encouragement of mathematical analysis and discourse, we offer a glimpse into the classrooms of 2 teachers. First we present an example from Teacher D's second videotaped lesson. Teacher D's classroom climate was consistently rated 1 with respect to analysis and discourse (i.e., an absence of both mathematical analysis and discourse). In this excerpt, students in Teacher D's class (C) were engaged virtually the entire period in a teacher-led (T) recitation of counting to 100.

T: All right, we're going to count our numbers from 1 to 100. All right now, looking over here [at number chart on the board], let's begin.

C: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 . . .

T: Okay we're going to stop. And, let's continue.

C: 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

T: We're going to stop. Whenever I put my hand up that means stop.

C: 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 [students and teacher continue counting in this format up to 100].

T: Notice 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. But all the numbers have a zero behind them. Did you notice that when you were counting?

C: Yes.

T: Very good. What is this number?

C: 20.

T: And this one?

C: 30, 40, 50, 60, 70, 80, 90, 100.

T: All right. I want you to take a look at this row. I want you to notice that in this row, the number 1 is repeated. As you go down, it's repeated. Repeated means you see it over and over. Look at this row. The number 2 is repeated. When we count, the numbers make a pattern. And what I am doing is I am showing you the pattern. I want you to listen to me count the first row. You just listen. 1 one, 2 ones, 3 ones, 4 ones. Are you listening?

C: Yes.

T: Okay, then your mouth is closed.

Table 1
Mean Ratings (and Standard Deviations) for Mathematical Analysis and Mathematical Discourse, by Teacher

	Teacher					
Climate measure	A	B	C	D	E	F
Mathematical analysis	1.29 (0.13)	1.06 (0.13)	2.71 (0.71)	1.10 (0.11)	1.83 (0.25)	1.03 (0.06)
Mathematical discourse	1.03 (0.06)	1.00 (0.00)	2.14 (0.46)	1.03 (0.06)	1.33 (0.08)	1.00 (0.00)

Note. For purposes of referring to specific teachers, we alphabetized the last names of the participating teachers and then labeled the teachers A to F in descending alphabetical order.

Although the students were engaged and actively participating, we found no instance of either mathematical analysis or mathematical discourse in this exchange, which was typical of the whole-class instruction that took place during the 51-min lesson. This scripted, predetermined lesson format left little room for student explanations or student ideas. Teacher D offered some analysis of the relationships among the ones, tens, and hundreds places in this and other lessons, but never drew on the students in these analyses.

As we examined Teacher C's lessons, which were rated markedly higher than those of the other teachers for both analysis and discourse, we learned how these features could skillfully and naturally be incorporated in first-grade lessons. We provide an excerpt from the second observed lesson. Earlier in the lesson, the students worked in small groups to write down everything they knew about 11. After finishing the small-group exercise, the students presented their ideas to the whole class. The teacher (T) elicited students' (S) work by asking them to select a representation of 11 that would "blow everyone away." Occasionally, she asked students to explain their ideas to the class:

T: How about over at Table 5? Which one do you have that you think is going to blow everyone away?

S: 20 take away 9 is 11.

T: Whoa! They did a subtraction problem. 20 take away 9 equals 11. Whoa! Maybe we should ask them to prove that to us. How did you come up with that answer? 20 take away 9?

S: [Name] did it.

T: [Name], how do you know that for sure? Can you tell us what strategy you used to figure that out?

S: Because we made numbers. 14 like take away 3 . . ..

T: Oh, you tried all different things until you came up with the answer. Good, cause that's exactly right!

Although we note that the teacher was clearly in charge and orchestrated student input, she asked the students for mathematical analysis and they complied by participating in mathematical discourse. Thus, we have evidence that first-grade students can be engaged successfully in coming up with creative solutions and in presenting and defending their ideas to others. Further, we have evidence that regular classroom teachers provide opportunities for their students to develop mathematical authority.

Specific Teacher Practices Communicating Mathematical Authority

In our next analyses, we focused on the three variables of specific instructional behaviors, which we developed to understand how teachers directly communicated a sense of mathematical authority to students: teacher questioning practices, the source credited to verify the legitimacy of mathematical information, and teachers' appropriation of student ideas.

Teacher questioning. We focused on three basic types of questions: answer known, requests for explanation, and requests for student ideas, consistent with previous research (e.g., Lampert, 1990; Perry, VanderStoep, & Yu, 1993; Stigler et al., 1996; Yackel & Cobb, 1996). These question types are defined and exemplified in Table 2. On average, teachers asked 68.57 questions ($SD = 31.75$) in a single lesson. On average, rates of answer-known questions were high across teachers (91.5% of questions). Only a small proportion of questions were requests for explanations (4.3% of questions) or requests for student ideas (4.2% of questions). The proportion of each of the question types asked by the 6 teachers is shown in Table 3.

We were interested in the extent to which teachers and their use of particular types of questioning were associated. Using the Pearson chi-square statistic, we rejected the hypothesis of independence between teachers and question types, χ^2 (10, $N = 6$) $=$ 414.92, $p < .001$, indicating a dependency between certain teachers and certain questions. To investigate the nature of the dependency between teachers and type of questioning, we fit a row–column (RC) association model (Agresti, 1990; Clogg & Shihadeh, 1994) to the data, using the computer program LEM (Vermunt, 1997). The RC association model is an extension of a log-linear model for two variables, in which the interaction between the variables is represented by the product of category scale values, which are estimated (along with other parameters of the

Table 2
Summary of Coding System for Types of Questions

Question type	Description	Example
Answer known	Questions for which there is a single correct answer or those that are clearly designed to elicit a single correct response such as a request for a definition established previously by the teacher.	"How many 10s are in 15?" "What does *even* mean?" (Seeking the answer, "it has a partner" that had been stated earlier in the lesson.)
Request for explanation	Requests for an explanation of a solution or for mathematical evidence to support a mathematical idea.	"I think we're going to have to ask you to tell us how you came up with that solution." "How do you know that for sure?"
Request for student ideas	Requests for student perspectives on possibilities such as a solution that might work to solve a problem; these requests were typically used to frame a problem.	"Tell me one way that you could show 11."

Note. We used the context of the lesson to guide our interpretation for the category code for specific questions.

Table 3
Proportion of Types of Questions (and Standard Deviations) Asked by Teachers

	Teacher					
Type of question	A	B	C	D	E	F
Answer known	.97 (.03)	.97 (.04)	.71 (.15)	.99 (.01)	.92 (.04)	.96 (.05)
Call for explanation	.02 (.03)	.03 (.04)	.11 (.05)	.00 (.00)	.07 (.04)	.04 (.05)
Call for student idea	.01 (.01)	.01 (.01)	.18 (.11)	.01 (.01)	.01 (.01)	.00 (.00)

Note. For purposes of referring to specific teachers, we alphabetized the last names of the participating teachers and then labeled the teachers A to F in descending alphabetical order.

model) from the data. Although the RC model fit considerably better than the model of independence (i.e., likelihood ratio statistic for RC model, $G^2 = 18.46$ (4), $p = .001$, vs. the independence model $G^2 = 247.20$ (10), $p < .001$), the lack of fit of the RC model is statistically large. However, the dissimilarity index (Agresti, 1996), which equals .008, indicates that the RC model fits the data well in a practical sense; that is, the model captures most of the structure in the data. A dissimilarity index equal to .008 means that only 8% of the observations (i.e., 17 observations out of the total 2,127) would need be moved from one cell to another for the model to fit perfectly. Furthermore, all the standardized residuals, given in Table 4, are relatively small except for one (Teacher D and call-for-student-idea questions).

Further analyses assist in interpreting this dependency. The estimated scale values for each of the teachers and each of the question types from the RC model are plotted in the top and bottom portions, respectively, of Figure 1. The smaller the relative difference between the scale values for teachers, the more similar the teachers are in terms of their odds of using one question type versus another. Teachers B and D and Teachers F and A are relatively similar to each other in terms of the odds with which they use particular question types. Teacher C stands out as different from her peers if we use the fact that (the modeled) odds ratios equal the product of differences between scale values for teachers and differences between scale values for question types. Although all teachers are likely to ask answer-known questions (see Table 3), the odds that Teacher C asks for student ideas versus either explanations or known answers is greater than the odds of asking this question type for any other teacher. Furthermore, the odds that Teachers B or D ask answer-known questions versus either of the other question types are greater than the odds for the other teachers.

Table 4
Standardized Residuals in the Log-Linear Model for Teacher Question Types

	Teacher					
Question type	A	B	C	D	E	F
Answer known	0.011	−0.005	0.199	0.077	−0.200	0.005
Call for explanation	−0.138	0.145	−0.975	−1.857	1.937	−0.078
Call for student idea	0.128	−0.325	0.350	3.288	−1.378	0.084

Note. For purposes of referring to specific teachers, we alphabetized the last names of the participating teachers and then labeled the teachers A to F in descending alphabetical order.

By drawing on the classroom dialogues, we can understand how different types of questioning techniques were situated within a lesson. It was common in the classrooms of Teachers A, B, D, and F for the majority of the lesson to involve rapid answer-known questioning by the teacher. Here is an excerpt from Teacher B's (T) first lesson to the students (S):

T: [Name], tell me the next two numbers [on the worksheet].

S: 55 and 75.

T: Which is larger?

S: 75.

T: Good for you, circle 75. [Name], next box.

S: 84 and 85.

T: Which is larger?

S: 85.

T: Last box, [Name], tell me the two numbers.

S: 79 and 99.

T: Which is larger?

S: 99.

T: Good. Let's look at page 160. Now, we're looking at what number is . . .

S: Tinier, littler.

In contrast, although Teacher C (T) presented plenty of answer-known questions, calls for explanations and student (S) ideas routinely were embedded in the flow of the lesson. Here is an example from her second lesson:

T: All right, let's take a look at the very first math problem, the sample that's already done for us: 8 + 3. Let's talk about some ways that good math students do that. Raise your hand if you can give us a strategy. A way that you would do that problem. [Name], how would you do 8 + 3?

S: 8 + 3?

T: What could you do? What could a good math student do to get that answer?

S: 8 and count 3 more.

T: Okay, take 8 and count on 3 times. 8, 9, 10, 11. That's one way you can do it. Does anyone else have another way you can do it? [Name]?

S: 8 + 3.

T: All right, 8 + 3. Are you using the counting-on method also? Okay, I think that's a favorite method in this class, counting on. What if

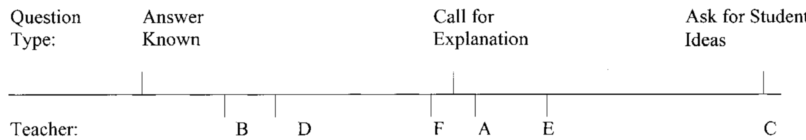

Figure 1. The estimated scale values for question types are plotted against teachers' use of these question types. The scale values for the question types are −.71 (Answer Known), −.001 (Call for Explanation) and .71 (Ask for Student Ideas). The scale values for teachers are −.52 (Teacher B), −.41 (Teacher D), −.05 (Teacher F), .05 (Teacher A), .23 (Teacher E), and .71 (Teacher C).

you're not comfortable counting on? I know you are, but what's something else that a person could do besides counting on?

S: Keep the higher number.

T: Keep the higher number in your head and count on. Can you use counters?

S: You could use a number line.

T: You could use a number line. Now, there's one problem with the number line on your desk, isn't there? [Name], what's one problem with the number line on your desk?

S: It doesn't go up to 11.

T: It doesn't go up to 11, does it? So what would be something you could use instead of the number line on your desk? . . . [Name]?

S: What about up there?

T: Or use the giant number line up on the wall. All right, there's lots of things you can do to help yourself, always remember that. Do whatever you need to help yourself.

As we see from this example, the discourse in Teacher C's classroom was focused on owning the process of solving a problem, in the sense that the teacher wanted to know what the students could do to solve the problem. Further, by asking students to generate ideas for solving problems or to provide explanations for their solutions, Teacher C emphasized that there are multiple pathways to mathematical solutions that lie within her students' repertoires.

Sources of verification. In Initiation–Response–Evaluation (IRE) sequences (see, e.g., Mehan, 1979), initiations or questions, and the ways in which responses to those questions are evaluated have the potential to communicate to students that they have mathematical authority. For example, if the teacher allows the whole class to verify a classmate's answer, the teacher grants authority to the whole class. Thus, we coded who was allowed to judge the correctness or legitimacy of students' responses. Three sources of verification were possible: the teacher, the math community (i.e., the students in the classroom), and the mathematics text or mathematical convention. These sources of verification are described and exemplified in Table 5.

On average, 96.5% of student responses were verified by teachers, 3.5% were verified by the classroom community, and none were verified by mathematics as a discipline. Many instances in which students (S) were the source of verification involved the class (C) as a whole determining the correctness of a peer's

Table 5
Summary of Coding System for Who Is Permitted to Verify a Student Response

Who verifies?	Description	Example
Teacher	The teacher: (a) verbally or nonverbally evaluated the student's response; (b) repeated the question to another student following an incorrect response; (c) in the case of questions following in a sequence, continued the sequence of questions to indicate a correct response.	(a) T: 5 + 5 + 1 = 11. Whoa! Very good! (b) T: How many children now on the table? S: Nine. T: How many children now on the table? (c) T: 15 is how many tens and ones? 16 is how many tens and how many ones? Etc.
Math community	The teacher designated or permitted a single student or multiple students to verify the legitimacy of a peer's response.	"Is she right?" "Does everyone agree?" "Let's count it out together to see if he's right."
Mathematics text or mathematical convention	The teacher deferred verification to mathematical convention or to the text as a representative of the discipline.	"On the triangle they are going to call this the base and they are going to call this the height" (Stigler et al., 1996, p. 170).

Note. T = teacher; S = student.

response, often along with the teacher. For instance, from Teacher F's (T) classroom:

> T: [writing 17 hash marks on the board] Now I want you to look over here and tell me how many. [Name]?
>
> S: 15.
>
> T: Not 15. [teacher verifies incorrect answer] [Name]?
>
> S: 17.
>
> T: 17?
>
> S: Uh uh, 16.
>
> T: Let's count together.
>
> C: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17.
>
> S: [Name] had it right.
>
> T: Right.

We examined whether teachers showed similar patterns of verification. Using the Pearson chi-square statistic, we rejected the hypothesis that teachers used similar patterns of verification, $\chi^2(5, N = 6) = 19.98$, $p = .001$. This suggests a dependency between teachers and sources of verification. Teacher D used teacher verification essentially exclusively (99.6% of the verifications), whereas all the other teachers used teacher verification about 95–98% of the time.[3] Despite this significant effect, review of the percentage of verification granted to teachers clearly indicate that for practical purpose, all 6 teachers established and maintained themselves as the source of verification of mathematical responses and ideas, nearly to the exclusion of any other source.

We were further interested in relationships between the type of question asked and the source used to verify the response to the question. In particular, we wanted to know if teachers were more willing to encourage students to verify responses to answer-known questions, compared with responses to requests for ideas or explanations. Because we observed too few instances in most categories to perform statistical tests, we limit our report to descriptive findings. Of the opportunities that students had to verify the legitimacy of a response, 92.0% (58 instances) were generated by an answer-known question, 3.1% (2 instances) involved verification of a mathematical explanation, and 4.9% (3 instances) involved verification of a mathematical idea. The mathematical information that students were most frequently granted the opportunity to validate were numerical facts. In most cases, students validated peer or teacher responses with a yes or a no (e.g., Teacher: "Is she right?" Class: "Yes"), rather than by offering a mathematical explanation themselves.

It is noteworthy that in only one instance did a student challenge a response or idea of a peer before the teacher asked for a verification of that response. In other words, at only one point in 30 mathematics lessons did a student assume mathematical authority without an overt solicitation from the teacher. We believe that the predominant instructional arrangements of teacher-led recitation or (less frequently) teacher-led discussion of ideas set the stage for the teacher to be the one who determined which source would generate and validate mathematical ideas.

Appropriation of student ideas. If teachers acknowledged or integrated students' explanations or ideas, in essence, teachers were crediting students with the authority to contribute substantively to the mathematical discourse. Because instances of students volunteering ideas were absent, instances of appropriation arose when students offered ideas in response to a teacher's question. With the exception of one category, teachers' appropriation of students' ideas and explanations could fall into one of several mutually exclusive categories. The teacher could (a) dismiss or reject the contribution, (b) acknowledge that there was a contribution but not incorporate it into the lesson, (c) acknowledge the contribution by merely restating it, (d) probe the student further for clarification, or (e) use the idea to make connections to other mathematical content. The one, nonmutually exclusive category was coded when the teacher also acknowledged the student who offered the contribution by crediting that student with the idea (Lampert, 1990; Stigler et al., 1996). These categories are described and exemplified in Table 6.

We were interested in what teachers did with the ideas and explanations that their students generated.[4] Of the 108 student ideas or explanations, only 1% were dismissed or rejected. Teachers primarily responded to students' ideas and explanations by restating the student's response without altering the content: 63% of appropriations fell into this category. Teachers occasionally (for 18% of students' explanations and ideas) followed with a probe for clarification. These instances most typically occurred in response to an incorrect or incoherent idea or explanation. And, teachers (for 13% of students' explanations and ideas) essentially did nothing by only acknowledging the students' response and did nothing to incorporate the response into the lesson. In only a few instances (for 5% of students' explanations and ideas), the teacher connected a student's response to other mathematical concepts or redirected the classroom activity to develop a student idea or explanation.

In addition to examining what teachers did with student responses, we explored how frequently teachers attempted to maintain a connection between an idea or explanation and the student who advanced it. Ideas and explanations were rarely linked to students: In only 5% of all the appropriation instances did this occur.

Despite their infrequency, it is instructive to examine instances of appropriation to illuminate how teachers can successfully grant authority to students through this practice. In Teacher C's class, after students had gone through and provided their favorite ideas and explanations about the number 11, Teacher C (T) concluded the lesson with the following exercise based on students' ideas:

> T: All right, 12 take away 1 equals 11. Something tells me that we could go on and on and on and think of all other ways. Let me tell you what I saw at some of your tables. I saw some tallying being done; I saw 11 tallies. I saw pictures, I saw 11 balloons being drawn; That's another way to make 11. I saw lots of addition equations for 11. . . . But you really showed me that you learned a lot about 11. . . . Now as I look at these papers, I see something from Table 1 that is extremely interesting and I'd like to talk about it. Table 1 came up with a very interesting way to tell 11. They

[3] As further confirmation of this finding, we reanalyzed the data excluding Teacher D's data and found that, without Teacher D, we could not reject the hypothesis of homogeneous distributions for the remaining teachers, $\chi^2(4, N = 5) = 4.15$, $p = .39$.

[4] We did not examine teacher differences in appropriation rates because our previous analyses had indicated that Teacher C had a disproportionate share of the appropriation instances. Instances of appropriation for the other teachers were too infrequent in number for analyses.

Table 6
Summary of Coding System for Appropriation of Student Ideas

Type of appropriation	Description	Example
Dismiss or reject student contribution	The teacher dismisses or rejects a student's contribution.	T: Can someone tell me or think of something that comes in a group of 10? S: 11. T: No. [Teacher moved on with lesson.]
Acknowledge but not incorporate	The teacher explicitly acknowledges a student idea by acknowledging the student's response but does not draw on the student idea to advance the lesson in any way.	The teacher commented "good" and moved on with the lesson.
Acknowledge by restating student ideas	The teacher restates a student idea to make it more clear without altering the substance of the idea.	T: Why do we know that's 19? S: Because it has 9 ones and 1 ten. T: 1 ten and 9 ones.
Probe for clarification	The teacher acknowledges and asks for clarification of a student's idea. Note that this was typically coded when a student offered an illegitimate or incoherent idea.	T: Who do you think the last person standing will be? Why? S: Because you're not by them. T: Because I'm not by them?
Make connections to other mathematical content	The teacher uses an idea presented by a student to make a connection (i.e., introduce, bridge to, or further) the development of mathematical concepts.	T: How would you do 8 + 3? Can you use counters? S: You could use a number line. T: Now there's one problem with the number line on your desk, isn't there? S: It doesn't go up to 11. T: It doesn't go up to 11, does it? . . . We have other number lines here.
Idea connected to student	The teacher explicitly links an idea to the student who offers the idea.	T: Marquisha had a great approach to solving this problem that I want us to follow up on.

Note. T = teacher; S = student.

> started right away by looking for patterns. . . . You're going to watch what I put on the overhead and I'm going to be writing something down, and if you think you've discovered the pattern, raise your hand, just keep it up there, and we're going to try and see if everyone can figure out what the pattern is by the time I get finished. And I got this idea from Table 1.

In this sequence, Teacher C used Table 1's idea as a bridge to help the class understand the relationships among numbers between 1 and 20. Interestingly, despite repeatedly crediting Table 1 with this idea, we note that Teacher C never involved those students in explaining their idea to the mathematical community.

One additional example from Teacher C's class illustrates a mundane activity that was transformed into an act of recognizing students as contributors who deserved credit. As Teacher C passed out math worksheets, she told her students to put "your name at the top so you get credit for all that hard work, all that good work." The point here is that all the students in all of the math classes we observed were instructed to put their names on their worksheets; it was only the students in Teacher C's class who had this framed as an opportunity to be credited for their good and hard work.

Incidental Comments That Communicated Ideas About Mathematical Authority

One final issue relevant to mathematical authority became apparent as we reviewed videotapes and transcripts of the 6 teachers' lessons. In particular, we noted that teachers often made comments to students that were not captured by our coding categories, yet appeared to credit mathematical authority to various sources. Our analyses thus far have focused on measures of mathematical analysis and mathematical discourse for the lesson and aspects of teacher–student interaction from an instructional standpoint. Yet, less formal elements of discourse found in the classroom may also communicate critical information about principles of social order within a culture (Ochs, 1986).

We found that incidental comments communicated attributions of mathematical authority in two ways. First, we noted that teachers often used language to suggest that they were granting mathematical authority to students, but followed with actions that betrayed their words. For example, Teachers A, B, D, E, and F were fond of saying to the class, "Let's do it together" or "Help me do this," but followed such statements with activities that were entirely scripted by the teacher and verified for accuracy and legitimacy by the teacher. Although these comments were uttered, these same teachers made explicit comments to students that clearly positioned the teacher as the sole authority and the students as passive participants in learning mathematics. For example, Teacher D, at the end of Day 2 following an exercise of counting to 100 and learning about place value, applauded her students and thanked them for being such a "good audience." Labeling students as the audience places them as passive voyeurs and the teacher as the actor with control of the situation.

Second, although the domain of mathematics was never credited with the authority to evaluate students' contributions (e.g., we never witnessed teachers claiming authority based on what the book tells us), teachers at times distanced themselves from the domain of mathematics by referring to the text with deference. For example, Teachers B, D, and F in particular had comments peppered throughout their lessons such as: "they show us," "they want us to," and "they tricked us." These comments situated the teacher and her students as outsiders in the community of mathematical thinkers.

These examples stood out to us because the messages they communicated seemed unambiguous. Our point is that the degree to which children seriously take themselves and their peers as participating members of a mathematics community is likely to be influenced by the incidental and routine comments that teachers make about their own place within the discipline of mathematics.

Discussion

In this investigation, we asked whether, and if so how, ordinary first-grade teachers granted authority to their students and created classroom communities in which students participated in mathematical analysis and discourse. In essence, we found that of our 6 teacher participants, only 1 granted authority to her students and created a classroom community in which students participated in mathematical analysis and discourse. And although we found that this teacher used practices that invited students to assume responsibility as members of a mathematical community, her use of these practices did not saturate her lessons; the majority of this teacher's behaviors still reinforced herself rather than the classroom community as the source of mathematical authority. Still, in contrast, except for isolated, brief instances, all of the other teachers' observed lessons were bereft of behaviors supporting these principles.

From these findings we make three interrelated points. Our first point centers on how aspects of reform ideals realistically manifest themselves in everyday mathematics classrooms. The second point focuses on the depth of teachers' commitments to reform-based practice in mathematics. And our third point addresses possible implications of our findings for students' learning of mathematics. In the remainder of the discussion, we explain each of these conclusions.

Mapping a Middle Ground for Regular Classroom Teachers

Some researchers (e.g., Cobb et al., 1992; Lampert, 1990) have portrayed teachers who might be identified as adhering to either reform or formalist positions, but not to both. Schifter's (1996) and the Cognitively Guided Instruction group's (e.g., Fennema et al., 1996) teacher narratives illustrate the processes that teachers undergo in transition from traditional to reform-oriented practice. Yet, all the teachers portrayed by these research teams share in common an affiliation with—and correspondingly, support and guidance from—educational researchers. We embarked on our investigation uncertain where regular and uncoached teachers might be situated with respect to these issues. We found that some of our teachers could be aptly characterized as formalist. Five of the 6 teachers in the present study displayed only occasional and isolated behaviors that could grant mathematical authority to students. The vast majority of these teachers' practices firmly established and maintained the teacher as the source and authority of mathematical knowledge. However, on the basis of our findings for Teacher C, we suggest that at least some teachers naturally blend behaviors that both provide students with key concepts and procedures that are known and accepted in mathematics and also foster students' development as mathematical thinkers. Further, we specified precisely in what contexts and in what ways these natural moments were likely to occur.

We found that Teacher C occasioned multiple opportunities for her students to fashion a sense of mathematical authority. Students were often held accountable for their mathematical ideas, independent of whether these ideas evolved in whole-class, small-group, or individual seat work formats. What aligned Teacher C with a growth-and-change perspective were her regular invitations to students to offer their ideas and explanations for their thinking in all formats and in all lesson structures. As an example, recall that Teacher C had students work in groups to come up with multiple ways to represent 11 and then had the students share their ideas with the class. But Teacher C did not merely have students report their solutions; instead she framed the sharing of solutions by asking students to contribute an idea that would astonish the class. In this way, she credited first-grade students, even before hearing their solutions, with the ability to create mathematical ideas so powerful that "we would all be blown away."

Furthermore, across the 5 days we observed, Teacher C positioned herself and her students as members of a mathematical community. She regularly emphasized to students the importance of finding one's own way to approach mathematical problems and indicated that ideas and strategies were legitimate if evidence was available to support them. As an example of this practice recall that Teacher C requested student strategies even on an example problem that was already completed. Moreover, she made this request by asking how "good math students" would solve the problem, implying merely by asking for their input that her students were good math students. On occasion, Teacher C reflected on her own mathematical thinking and acknowledged publicly when a student advanced an idea that she had not considered.

Although Teacher C's instruction in many ways revealed growth-and-change underpinnings, we emphasize that her classroom did not approximate the kind of discourse community that Lampert (1990), Peterson (1993), and Stigler and Fernandez (1995, for Japanese teachers) have described. Teacher C relied heavily on answer-known types of questioning, she positioned herself rather than students as the ultimate source of verification of mathematical ideas, and she only infrequently used student ideas and explanations as a launching point for the development of new mathematical ideas. In other words, despite engaging in multiple practices that granted students a sense of mathematical authority, Teacher C rarely stepped away from the central role as classroom leader to allow true public discourse about mathematical ideas. We speculate that these are specific areas in which teachers may find it challenging to relinquish authority to the classroom community. Future efforts to bring teacher practice into alignment with reform ideals may be well advised to keep in mind that the pruning away of old practices may be difficult even while teachers are successful at implementing reform-minded practices (also see Cohen, 1990; Cohen & Hill, 1998).

Blending of Positions: Veneer Versus Substance

Our second conclusion extends the previous discussion to emphasize that it is not simply the absolute number or percentage of reform-minded behaviors that is central to children's development of mathematical authority. This is true in either direction: A small number of these behaviors may be adequate to make a difference if they are salient and not undermined by counter-productive behaviors. Likewise, the mere presence of reform-minded, growth-and-change-motivated activities is not sufficient because these behaviors can be undermined by other activities, practices, and behaviors that firmly communicate that the teacher is the sole mathematical authority in the classroom. For example, Teacher B frequently said "let's do this together," which implied that she and the class would coconstruct mathematical meaning. Even if this message had ambiguity in its intent to communicate a willingness to coconstruct mathematical meaning, Teacher B's actions had no ambiguity in relation to this point: She led and the students complied by carrying out prescribed procedures and filling in known answers. The point we wish to make is that teachers can use language and apply behaviors that, on the surface, stem from a growth-and-change perspective and ostensibly invite all children to participate, but such overtures are not enough: The words must be backed up with requests and formats for substantive student participation consistently throughout a lesson if we really are to witness students engaged with mathematical ideas and creating mathematical knowledge.

Our observations of the students in these first-grade classrooms give us confidence in this conclusion. In Teacher C's class, students readily elaborated on their thinking when called on to do so and quickly got down to business when assigned a small-group activity. We witnessed students huddling excitedly as a team to generate mathematical ideas to share with their larger classroom community. In contrast, on the infrequent occasions that other teachers solicited students' ideas, students appeared to be uncertain how to respond; teachers' attempts to accomplish collaboration in small groups was often met with student resistance and required significant amounts of transition time to establish. This suggests that individual teacher practices can conspire to build a classroom culture that supports or undermines student authority.

Implications for Emphases on Procedures in Mathematics Instruction

To be true to the discipline, mathematics instruction should involve a balance between having students learn what is already known and having students apply that knowledge to generate, analyze, support, and refute their own and others' mathematical ideas (Ball, 1993; Schoenfeld, 1986). Unquestionably, fluency with basic procedures of mathematics is central to mathematical understanding; simple skills such as those with which the observed students were engaged (e.g., counting on) must become automatized because they provide a foundation for more sophisticated algorithms (Mayer, 1985). And, as researchers (e.g., Hiebert & LeFevre, 1986) have pointed out, drill-and-practice techniques can be a useful instructional technique for the development of such skills. However, exclusive attention to procedure-based instructional practices likely presents a risk to students' mathematical understanding because attention to scripted procedures can interfere with the development of students' understanding of underlying concepts (Hiebert & Wearne, 1992; Perry, 1991). As an example, Perry (1991) found that if children were presented with procedures for solving problems—independent of whether other, more conceptual information was also provided—the students were likely to use that procedure and not to consider the concept underlying the application of the procedure. Moreover, students receiving procedural instruction did not make adjustments and transfer their new procedural prowess to novel problems (also Hiebert & Wearne, 1992). In the classroom observations that we analyzed for this investigation, with the exception of Teacher C's classroom, students were engaged almost exclusively in approaching learning with the expectation that everything they were to learn was already known and the students had few opportunities to move beyond learning simple procedures. Furthermore, they were given little opportunity to appreciate how and why these skills might apply to problem solving in mathematics. The implication of this is that although these students are likely to learn the mathematical procedures that they have been taught, it is significantly less likely that these students will learn that they can apply the procedures that they have mastered flexibly across mathematical topics.

In sum, we argue that, at best, in the mathematics classrooms we observed and potentially in many more like them, multiple forces conspire to send children mixed messages about their authority to develop and to verify mathematical knowledge. At worst, but perhaps most commonly, children learn very early in their schooling experiences that mathematics is a discipline already determined by others—and consequently, a discipline to which they have little to contribute. Even within this generally bleak landscape, we found a counterexample: One teacher seamlessly integrated her students into the production of mathematical knowledge. Although prior research has provided other such examples, these other examples often seemed idealized, perhaps because the teachers producing these examples were privy to unique circumstances, including collaboration with university research teams. From our research, we conclude that it is possible to see exemplary practices in action in regular mathematics classrooms, even in early grades. But perhaps it is not realistic to expect that teachers can find a balance between teaching what is known and truly socializing students to be mathematical thinkers without an intermediate step, much like we witnessed in Teacher C's classroom. We hope that Teacher C's classroom, and the discourse documented therein, may serve as an accessible model for teachers who are attempting to move their instruction toward the growth-and-change position, because her behaviors demonstrate natural moments for empowering students with mathematical authority, thereby enabling young children to become legitimate members in a mathematical community.

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