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Indexing Transitional Knowledge

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Various indexes have been proposed to mark when a child is on the verge of acquiring new knowledge. This article described a new method for indexing this point of transition, which is based on the specificity of children's verbal explanations. The degree to which children were specific in their verbal explanations was related to whether they benefited from instruction. In particular, children who were vague in their explanations (i.e., less specific) benefited from instruction more than children who were explicit in their explanations (i.e., more specific). This index provides insights into the child's mental state prior to acquiring new knowledge.

To understand cognitive development, one must have a good understanding of the processes of change and be able to characterize, as best as possible, knowledge in transition. As others (e.g., Berliner, 1987; Posner, Strike, Hewson, & Gertzog, 1982; Siegler, 1989; Siegler & Crowley, 1991; Sternberg, 1984; West & Pines, 1985) have suggested, a reasonable place to begin understanding processes of cognitive change is to investigate and document the points at which changes or transitions are most likely to occur (e.g., Thelen & Ulrich, 1991). Unfortunately, relatively little research has been conducted on this topic. The goal of the present research is to attempt to address this by exploring the utility of an index of transitional knowledge.

The transitional knowledge state has been characterized in several ways (see, e.g., Siegler, 1989; Sternberg, 1984). Many of these characterizations can be summarized in terms of stability of knowledge. What this means is that when knowledge is stable, it is unlikely to undergo change. Knowledge may be stable among novices, who are not yet ready to learn a particular concept, or among experts, who no longer need to learn the same particular concept (although the expert may be unstable with respect to some other concept). The major concern of the present research is with the earlier stages of knowledge acquisition. Novices with stable knowledge states should not be receptive to instruction (and experts with stable knowledge states should not need instruction), whereas novices with unstable knowledge states should be receptive. What we attempt to accomplish in the present research is to provide an index of unstable knowledge, with respect to children's acquisition of a mathematical concept. Our goal is to describe children's behaviors when they are in a state of transitional knowledge—as they are coming to acquire new conceptual knowledge—so as to understand better when and how cognitive change occurs.

Previous Characterizations of Transitional Knowledge

Perhaps the best known position on children's cognitive change is Piaget's (e.g., 1985). Piaget noted that children's knowledge is unstable when they are in a state of disequilibrium. Disequilibrium occurs when the child's current construction of the world no longer explains his or her experiences with the world. This disequilibrium provides the impetus for the child to restructure his or her current cognitive understanding so that this understanding is more consistent with his or her experiences with the world. Balance is accomplished through the child's accommodation of new information from the environment. In other words, equilibration is a state in which the world "makes sense" to the child. Although Piaget's theory has appealing aspects, it has been difficult to verify and to characterize when knowledge is in transition (e.g., Bell, Grossen, & Perret-Clermont, 1985).

Others have attempted to operationalize and characterize the transitional knowledge state in various ways. For example, Wilkinson (1982) suggested that children's transitional knowledge can be identified by inconsistency in problem solving. Inconsistency works well as an index of transition because children who are inconsistent demonstrate that they can entertain multiple approaches to one problem (by definition). According to Wilkinson, a child who possesses multiple approaches ought to be closer to making a conceptual transition than a child who does not, because it is likely that the child with multiple approaches will attempt to reconcile them and move to a new understanding or discard incorrect approaches and thus move to a higher level of understanding.

Strauss's (1972) claims are quite similar to Wilkinson's. Strauss argued that children with partial knowledge were in a state of transition. He argued that children who were able to conserve on some tasks but not on others (i.e., had a partial understanding of conservation) were more likely to be ready to make a transition than children without a partial understanding. It is easy to accept that children who possess a partial and thus more complete understanding of a concept would progress

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to a more advanced state of knowledge before children who do not possess a partial understanding.

Another characterization of the transitional knowledge state is one in which all of the necessary components exist, but these components are not integrated into a coherent representation. For example, Wagner and Torgesen (1987) hypothesized that children who have the component parts that are necessary to read but have not integrated them are in a state of transitional knowledge with respect to learning to read. Indeed, they found that children who possessed necessary components learned to read earlier than children who did not possess these components. A potential problem with using this approach more generally is that it is important to determine all of the components relevant to a task and then to determine whether the child possesses those components. For some concepts, defining these component parts or assessing knowledge of them may be problematic.

The work of Church and Goldin-Meadow (1986) provides us with yet another way of indexing and characterizing the state of transitional knowledge. They found that children who were in a state of transition with respect to the concept of conservation possessed an unstable understanding of this concept. Unstable understanding was operationalized as children's mismatches between speech and gesture, occurring during explanations of conservation judgments. They labeled children who produced many gesture–speech mismatches *discordant* and children who produced few or no mismatches *concordant*. For example, children who justified a conservation judgment (such as when water was poured from a glass into a dish, it no longer contained the same amount of water) by saying one thing (e.g., "the glass is taller") while indicating something different with hand gestures (e.g., the width of the glass with their thumb and forefinger) were more likely to benefit from minimal training in conservation than children who produced few or no gesture–speech mismatches. In a second study, Perry, Church, and Goldin-Meadow (1988) found that this sort of mismatch provided an accurate index of transitional knowledge of a different concept: mathematical equivalence. Perry et al. found that most of the discordant children made great gains in their mathematics problem-solving performance after a very brief instruction; very few of the concordant children made any gains after instruction. They argued that the display of mismatch between speech and gesture was a general index of the instability of knowledge and thus an index of a state of transition with respect to a concept. Perry, Church, and Goldin-Meadow (1992) argued further that the simultaneous activation of more than one notion, as demonstrated in gesture–speech mismatches, may be crucial for cognitive change (also see Alibali & Goldin-Meadow, in press; Case, 1985).

Verbal Specificity as an Index of Transitional Knowledge

The available indexes of transitional knowledge provide us with a window into how the child may represent concepts and into how new concepts are built. Our hope is that other indexes of transition may provide us with richer and more complete characterizations of the transitional knowledge state.

An alternative index of transition has been suggested but not yet investigated systematically (Caron & Caron-Pargue, 1976; Siegler & Jenkins, 1989; Wagner, 1987). In particular, it has been suggested that children become vague and inarticulate as they grapple with a new conceptual understanding. Siegler and Jenkins studied children's acquisition of a new strategy while they were solving addition problems. Anecdotally, they reported that the specificity of children's speech changed during their acquisition of a new strategy. They stated that children became "strikingly *less articulate* [italics added] on the trial on which they discovered the new strategy" (p. 108). During both the trial on which they discovered a new strategy and on the trial directly preceding it, children's speech was characterized by more pauses, incomplete sentences, and multiple starts and stops. Some of the children's answers provided little or no explicit information about the strategy they used. According to Siegler and Jenkins, "the clarity of children's self-reports was lowest precisely where we wished it to be the highest" (p. 108).

Siegler and Jenkins (1989) offered a few suggestions about why children were more inarticulate during these trials. First, they argued that generating a new strategy may usurp some of the mental resources necessary to monitor and produce an explanation. Second, children may be executing the new strategy while harboring the old strategy. Thus, their speech may have been vague because they were concentrating on more than one strategy at a time. Finally, they posited that in discovering a new strategy, the children had not yet developed a new vocabulary to describe their activity. However, it is difficult to determine which characterization is the most accurate. Therefore, Siegler and Jenkins's work supports the notion that changes in the specificity of children's speech can indicate cognitive change, but it cannot systematically confirm this relation.

Caron and Caron-Pargue's (1976) study of children's problem-solving ability provides additional evidence that children's speech can be examined to index cognitive change. Relevant to the present investigation, they found that the amount and type of verbalization produced during problem solving served as an index of the transitional knowledge state. The language the children used to describe the task varied as their ability to solve a mathematics puzzle progressed. Early in the learning sequence, children's language consisted of descriptions of their actions, with the use of many action verbs and few reflexive verbs. As children's problem-solving accuracy progressed, a decrease in the description of actions and an overall decrease in the precision of the language was noted. This coincided with the production of vague action verbs and an increase in the number of reflexive verbs. From this, it seems that features of verbal language may be used to index the transitional knowledge state.

Both Caron and Caron-Pargue (1976) and Siegler and Jenkins (1989) found that the specificity of children's speech could index changes in knowledge states, but they were unclear about how specificity was measured. For example, although Caron and Caron-Pargue's measure relied on production of vague verbs, they did not describe exactly how this measure was derived.

In the present research, we attempt to operationalize the notion of transitional knowledge through the verbal specificity of children's speech. We ask whether verbal specificity can be used to index transitional knowledge and, if so, how it may add to our understanding of the transitional knowledge state. Also, if we can identify the transitional knowledge state by using a measure of verbal specificity, this index must be compared with other known indexes of transition. For example, it will be important to examine where and how alternative indexes overlap so that the richest possible characterization of the transitional knowledge state may be derived. To address these issues, we first operationalize verbal specificity and then compare it with gesture–speech discordance on the verbal transcriptions of the same children analyzed by Perry et al. (1988). This is done for two reasons. First, the gesture–speech discordance index has already been shown to be a significantly good predictor of the transitional knowledge state for those children and for the concept tested by Perry et al. Second, given that our aim is to discover not only when cognitive change is likely to occur but also to broaden our characterization of the transitional knowledge state, it is helpful to compare indexes that may support different characterizations of transitional knowledge.

Method

Subjects

Thirty-six children (19 girls and 17 boys) from the fourth and fifth grades participated in this study. Overall, the mean age of these children was 10 years and 0 months. The mean age of the 19 fourth-grade children was 9 years and 6 months, and the mean age of the 17 fifth-grade children was 10 years and 6 months. The children attended five schools in an urban area. Although 40 children originally participated, only the 36 children who incorrectly solved all of the problems on the pretest and who provided verbal responses to all six pretest problems were included (3 children solved some of the pretest problems correctly, and 1 child did not provide verbal responses for all of the problems). We only included children who incorrectly solved all of the pretest problems in the present investigation to assure that all children initially appeared to have comparable knowledge of mathematical equivalence.

Materials

Problems were designed to test children's understanding of the concept of mathematical equivalence. Children were asked to solve addition problems similar to the following problem: $4 + 6 + 9 = \_\_\_\_ + 9$. Problems like this require a relational understanding of the equal sign, and often children do not solve the problems correctly without this understanding (Baroody, 1983; Behr, Erlwanger, & Nichols, 1980; Herscovics & Kieran, 1980). The pretests, posttests, and transfer tasks were all devised on the basis of this concept, Materials used in these tasks can be found in the Appendix.

Procedure

We tested each child individually and videotaped portions of the session. All children received a pretest, instruction, a posttest, and a transfer test. The procedures for these tasks are described here briefly; additional detail can be found in Perry et al. (1988).

Pretest. The children were asked to solve six problems on a paper-and-pencil test on their own (see the Appendix). After completing the paper-and-pencil test, children were asked to explain their solutions to a female adult at a blackboard. The adult wrote the problem and the child's solution on the blackboard and asked the child to explain how he or she solved the problem. After the child finished explaining the solution, the adult put the next problem on the blackboard. The child explained the solutions to all six problems from the pretest.

Instruction. Following the pretest, the child was given information about the goal underlying the problems. The instruction was administered by an adult female experimenter who was not present at the pretest and who was unaware of the child's understanding of equivalence.

The instruction began with the experimenter writing an addition problem of the sort that was on the paper-and-pencil test on the blackboard. With the new problem on the board, she told the child: "The goal of a problem like this is to find a number that fits in the blank that makes both sides equal; that is to make this side (pointing to right side of equation) equal to this side (pointing to left side of equation)." Children were allowed to ask questions and, if anything remained unclear, the experimenter repeated the instructions. However, no procedural instruction was given for achieving this goal, and no solutions were provided.

After showing the first problem, the experimenter wrote a second problem on the blackboard and asked the child to attempt to solve this problem. The experimenter gave no feedback to the child (other than general encouragement) but would repeat the instructional phrases if the child requested assistance. Up to this point, the child had heard the experimenter give instruction for one problem and was allowed to solve one problem. The experimenter repeated this sequence with two more addition problems of the same type. In other words, the experimenter gave the same instructions, without allowing the child to solve the problem, and then gave the child a new problem to solve. Thus, the child heard instruction at least twice and solved two problems.

Posttest. Immediately following the instruction, each child was given a paper-and-pencil posttest consisting of six addition problems similar to those on the pretest. After completing these problems, the child was asked to explain the solutions to the problems at the blackboard to the same adult who was present during the pretest. The administration of the posttest was identical to the pretest except for the problems used.

Transfer. A transfer test followed the posttest to assess whether the children could extend their understanding of mathematical equivalence beyond the addition problems on which they were instructed. This session was conducted in an interview style at the blackboard with the adult who previously had instructed the child. Each child was asked to transfer his or her knowledge to multiplication (e.g., $2 \times 4 \times 3 = \_\_\_\_ \times 3$) and to new forms (e.g., to judge whether $4 + 6$ was a correct solution for the problem $4 + 6 + 3 = \_\_\_\_ + 3$).

Coding of Success on Posttest and Transfer Test

The number of correct answers on the posttest and transfer test was recorded. As in Perry et al. (1988) and Perry (1991), children had to demonstrate that they did not understand the principle of equivalence before instruction and that they understood this concept after receiving instruction for us to judge them as having made a transition in their understanding. To be judged as having been in transition during the pretest, children had to make significant improvement on both the posttest and transfer test. Our criterion for significant improvement was that they answer correctly at least 5 out of the 6 posttest problems and 11 of the 12 transfer test problems. Children who did not solve any of the problems correctly on the posttest or transfer test were classified as not having succeeded after instruction and thus were evaluated as having not been in transition during the pretest.

Three of the children demonstrated partial success after instruction. Two of these 3 children correctly solved the posttest problems but failed to solve correctly some of the transfer problems (one failed the multiplication problems, and the other failed to transfer to new forms). The other child solved only four of the six posttest problems correctly and failed to transfer to new forms. We decided to classify these children with the others who had not demonstrated any success on the posttest or transfer problems for both empirical and theoretical reasons. Empirically, 92% of the children either solved all or none of the problems correctly, indicating that patterns of successes generally followed a bimodal distribution. Theoretically, the 3 children (8%) who were unable to solve problems on the pretest but were able to solve similar problems after training demonstrated difficulty transferring their understanding of mathematical equivalence to novel but related problems, which indicated that they had not made a complete transition.[1]

Coding Pretest Problems for Strategies

The children's explanations of the six solutions were coded for the strategy used to derive the answer. The coding of any particular explanation was determined by which numbers in the problem the child would have needed to obtain the solution that he or she had written. For example, if in the problem 4 + 6 + 9 = ____ + 9 the child put 28 in the blank, it was assumed that the child needed to add the 4, the 6, and the two 9s to obtain the solution of 28. For those rare cases when the explanation did not appear to lead to the written solution, we still based our coding of that problem on the child's written solution, because the task for the child was to explain how each written solution was derived.

Four different types of nonequivalence strategies were used by the children to solve the problems. The most common of these strategies was Add-All, in which the solution was derived by adding all of the addends, used on 57% of all problems. Another frequently used strategy was Add-to-Equal Sign, in which the solution was derived by adding all of the addends to the left of the equal sign, used on 29% of all problems. Another distinctive strategy was Add-to-Blank/Add-All, in which the solution was derived by adding all of the numbers up to the blank, putting that answer in the blank, and then adding together the two numbers on the right side of the problem and putting a second answer to the right of the problem (e.g., 4 + 6 + 9 = <u>19</u> + 9 = <u>28</u>). The remainder of the solutions (2%) was derived by idiosyncratic strategies (e.g., for 4 + 6 + 9 = ____ + 9, the child put 6 in the blank, presumably placing the 6 from the left side of the equation into the blank). Because these were so rare and because the five solutions that were classified as idiosyncratic seemed to be derived from different idiosyncratic approaches, these five solutions were omitted from further analysis. Descriptions and examples of the strategies used on the set of problems on the pretest can be found in Table 1.[2]

Coding Pretest Explanations for Verbal Specificity and Discordance

Explanations were defined as all of the information provided by the child in response to the experimenter's request for an explanation for the pretest solutions. Both verbal specificity and discordance were assessed on the basis of the coding of the verbal explanations of the pretest problems. Discordance assessment also required coding of the gestural explanations of the pretest problems. Only the coding of verbal specificity is described here. Information about the coding of discordance can be found in Perry et al. (1988).

First, we assessed individual explanations for degree of specificity, then we computed the average specificity of the six explanations. Individual specificity scores were derived by assigning one point for each of the numbers and operators (such as "and" or "plus" to indicate addition or "is" or "comes to" to indicate summing) mentioned in the explanation. For example, in the problem 4 + 6 + 9 = ____ + 9, if the child gave 19 as his or her solution and said "4 plus 6 plus 9 equals 19," this explanation would receive a score of 7 (i.e., one point for each of the seven elements mentioned).

Because different strategies require different amounts of explanation, the verbal explanations were standardized according to the strategy coded from the paper-and-pencil test. The standardization was based on the amount of information needed to adequately explain a solution. For an explanation to be considered explicit, the addends, the operators, and the final answer needed to be mentioned; otherwise the explanation was considered vague. The standardization values varied, depending on the solution to be explained. Thus, to be considered an explicit explanation of an Add-All solution, the child's answer needed to include 9 elements; to be an explicit explanation of an Add-to-Equal Sign solution, the child's answer needed to mention 7 elements; and to be an explicit explanation of an Add-to-Blank/Add-All solution, the child's answer needed to mention 11 elements.

Here are three examples. Three different children each solved the problem 4 + 6 + 9 = ____ + 9 by placing a 28 in the blank. The first child explained this solution by saying "4 plus 6 plus 9 plus 9 is 28." In this explanation, the child mentioned nine elements: the four numbers that were in the problem (4, 6, 9, and 9), three additive operators (plus, mentioned three times), one relational operator (is), and the solution (28). This explanation was standardized to 1.00 (i.e., 9/9) and was considered explicit, because the child left no question about what he did to solve this problem.

As a second example, a different child said, "Let's see, 4 plus 6, that equals 10, plus 9 more is 19, plus 9 more is 28." In this explanation, the child mentioned 13 elements: the 4 numbers that were in the problem (4, 6, 9, and 9), 3 numbers that were not in the problem (10, 19, and 28), 3 additive operators (plus, mentioned three times), and 3 relational operators (equals, is, and is). This explanation was standardized to 1.44 (i.e., 13/9). This explanation was also considered explicit, because the child provided both all of the necessary information to understand how he solved the problem and additional information (i.e., partial sums).

As a third example, a different child said, "I added those to get 18, plus 10 is 28." This child mentioned seven elements: three numbers not in the problem (18, 10, and 28), two additive operators (added, plus), and two relational operators (get, is). The child's explanation was standardized to .78 (i.e., 7/9). Although it does not take much imagination to figure out how the child arrived at the numbers mentioned in the solution, this child did not explicitly state how the solution and the partial sums were derived. We consider this sort of explanation to be vague, because critical information was omitted from the explanation.

Although children's gestures may have disambiguated some of their speech, for the reasons stated previously, we decided to rely on the children's speech alone to assess potential transitional knowledge. Indeed, the fact that the speech may not reveal what the child did to solve

[1] To make certain that we were justified in including these 3 children with those who did not demonstrate success after training, we also performed analyses with these children reclassified as demonstrating success after training. We found no difference in the significance of our results. In addition, when we omitted these 3 children from analyses, we found no difference in the significance of our results. Thus, the significance of the results (described in the Results section) does not depend on how these children were classified, and we left them with the children who did not demonstrate success after training because we remained unconvinced that they made a significant improvement in their understanding of the concept of mathematical equivalence.

[2] If an addition error was made so that the child's solution only differed by one or two numbers from a solution consistent with one of the pretest strategies, that strategy was coded. For example, in the problem 4 + 6 + 9 = ____ + 9, the child may have meant to add all of the numbers to the left of the equal sign but added incorrectly and put an 18 in the blank. In this example, the error would be noted, but the strategy would still be coded as Add-to-Equal Sign.

Table 1
Description and Examples of Strategies Used on the Pretest
Example problems: (a) 4 + 6 + 9 = ____ + 9 and (b) 3 + 6 + 8 = 3 + ____

Strategy	Paper-and-pencil solution	Description	Verbal example	Frequency of usage
Add-All	(a) 4 + 6 + 9 = 28 + 9	The child indicated that she added all of the numbers.	"I added 4 plus 6 plus 9 plus 9 and that equals 28."	57%
	(b) 3 + 6 + 8 = 3 + 20	Same as above.	"3 plus 3 is 6 and 6 more is 12, and plus 8 is 20."	
Add-to-Equal Sign	(a) 4 + 6 + 9 = 19 + 9	The child indicated that she added the numbers to the left of the equal sign.	"4 plus 6 plus 9 equals 19."	29%
	(b) 3 + 6 + 8 = 3 + 17	Same as above.	"I added 8 and 6 would be 14 plus 3 would be 17."	
Add-to-Blank/ Add-All	(a) 4 + 6 + 9 = 19 + 9 = 28	The child indicated that he or she summed the numbers up to the blank and added that sum with the remaining addend and a second sum.	"4 plus 6 equals 10, plus 9 equals 19 and 19 plus 9 equals 28."	11%
	(b) 3 + 6 + 8 = 3 + 20 = 23	Same as above.	"3 plus 6, that's 9, then plus 8 makes 17, plus 3 makes 20, and 3 plus 20 makes 23."	

the problem, nor even what the child may have intended to communicate, is what we are investigating here.

Each child's standardized scores were averaged across the six problems, resulting in a single specificity score for each child.[3] A child was classified as vague or explicit on the basis of his or her average standardized score. Averaged scores below 1.0 indicated that a particular child typically failed to mention all of the elements in the problem needed to reach the solution and that child would be classified as *vague*. Averaged scores above 1.0 indicated that a particular child mentioned more elements than was necessary for a clear answer and that child would be classified as *explicit*. Reliability for classifying children as vague or explicit was very high (92% agreement between raters).

Consistency of Verbal Specificity

It was possible that the average of the six responses would not accurately reflect the children's performance. If, for example, three of the explanations received high scores and three received low scores, then the average specificity score would be misleading. Thus, we examined the consistency of children's explanations in two ways. First, it was possible that children were not always consistent in providing all vague or all explicit explanations across the six problems in the pretest. The children who were classified as vague produced a mean of 4.50 vague explanations; the children who were classified as explicit produced a mean of 5.32 explicit explanations. Given that children were not consistently vague or explicit across all six explanations, we wanted to have an assurance that the average specificity score was indeed an accurate reflection of children's state of knowledge. In other words, we wanted to examine whether the mean scores might mask meaningful variability within the two groups. Thus, in a second measure of consistency, and to gain a sense of the variability of children's responses, we calculated and examined separately the standard deviations for the children classified as vague and the children classified as explicit. The standard deviations of the scores were 0.21 for the children who were classified as vague and 0.21 for the children who were classified as explicit. Therefore, the children who were classified as vague were not any more or less consistent in their specificity across explanations than the children who were explicit. Thus, we used the average specificity score to reflect a child's state of knowledge.

Results

Success After Instruction

Although none of the children solved any of the pretest problems correctly, the children differed in terms of their performance on the posttest and the transfer test. Of the 36 children, 39% succeeded after instruction, by our strict criteria.

We found no systematic relation between the children's pretest strategies and their postinstruction performance, $\chi^2(3, N = 36) = 1.79$, $p = .62$. This suggests that the transitional knowledge state is not marked by use of any particular strategy or strategies that children use before receiving instruction with respect to the concept of equivalence.

Verbal Specificity as an Index of Transitional Knowledge: Analysis of the Pretest

We chose to examine verbal specificity during the time when children were explaining their solutions to the pretest. This seemed to be the optimal phase of the children's participation for determining transitional knowledge, because this represented the time after children had the opportunity to work with the problems on their own (and note that none of the children included in this study indicated that solving the problems on their own promoted a transition because all 36 children solved all of the pretest problems incorrectly). This also represented the time before children were provided with any direct information that was intended to enable them to make a transition (i.e., before instruction was provided).

The children differed in how specific their pretest verbal explanations were. Average standardized specificity scores ranged from 0.37 to 1.49, with a mean of 1.06. Children were

[3] Recall that if the child had individual paper-and-pencil solutions that were coded as idiosyncratic, explanations of those solutions were not included in the calculation of the overall specificity score.

classified as vague if they had a mean specificity score below 1.0 (39% of the children—the range of scores was from 0.37 to 0.94, with a mean of 0.67) and as explicit if they had a mean specificity score above 1.0 (61% of the children—the range of scores was from 1.02 to 1.49, with a mean of 1.25).

Specificity scores were significantly and negatively related to learning outcome ($r = -.375$, $p = .02$). This suggests that the children who provided more specific explanations were less likely to succeed after instruction and children who provided less specific explanations were more likely to succeed after instruction.

Given the significant correlation between specificity and learning outcome, we wondered whether we would be justified in concluding that specificity acts as a continuous variable. To examine this, we performed two separate correlations: one for the children classified as vague, between specificity scores and learning outcomes ($r = -.13$, $p = .65$), and one for the children classified as explicit, between specificity scores and learning outcomes ($r = -.16$, $p = .48$). Given the insignificant relations between vagueness or explicitness and learning outcomes, we treated verbal specificity as a dichotomous variable for the remainder of our analyses.[4]

Considering verbal specificity as a dichotomous variable, we found that classification of verbal specificity was related to learning outcome, $\chi^2(1, N = 36) = 6.2$, $p < .02$ (see Table 2). Children who were classified as vague were significantly more likely to benefit from instruction than children who were classified as explicit. Nine of the 14 children who were classified as vague benefited from instruction, whereas only 5 of the 22 children who were classified as explicit benefited from instruction.

More important, verbal specificity was not related to individual differences among the children. We analyzed five variables that could have accounted for children's success after training, and none was related to verbal specificity. In particular, the groups did not differ on the use of pretest paper-and-pencil strategies, $\chi^2(3, N = 36) = 2.59$, $p = .460$; the groups did not differ in terms of children's grade in school (fourth or fifth), $t(34) = 0.259$, $p = .80$; the groups did not differ in terms of mean age, $t(34) = 0.91$, $p = .37$; the groups did not differ in terms of standardized achievement test scores, $t(34) = 0.46$, $p = .60$; and the groups did not differ in terms of gender, $\chi^2(1, N = 36) = 0.07$, $p = .79$. These analyses provide some confidence that children are vague or explicit with respect to their understanding of mathematical equivalence and not because they are somehow older, smarter, or different from each other in other obvious ways.

Verbal Specificity as an Index of Transitional Knowledge: Analysis of the Posttest

We examined posttest explanations of the children who did not benefit from instruction to determine whether children who were classified as explicit tended to be stable, even after instruction, and whether children classified as vague were relatively unstable in their approach to solving mathematical equivalence problems. In other words, we investigated whether explicit verbalizations were characteristic of a stable state and vague verbalizations were characteristic of a fleeting, transitional state. In general, children who were explicit before instruction remained explicit after instruction (76% of the 17 explicit children who did not benefit from instruction remained explicit); however, children who were vague did not demonstrate a consistent pattern after instruction. In general, children who were vague on the pretest demonstrated success in their posttest solutions (64% of the vague children).[5] Of the 5 vague children who did not demonstrate success after instruction, 1 remained vague, 2 became explicit, and 2 demonstrated partial understanding of equivalence. This analysis suggests that explicit verbalizations are characteristic of a stable state, whereas vague verbalizations are characteristic of a fleeting, transitional state.

Table 2
Proportion of Vague and Explicit Children Improving After Instruction

Postinstruction performance	Vague children (n = 14)	Explicit children (n = 22)
Success on posttest and transfer problems	.64	.23
No success on posttest or transfer problems	.35	.77

Comparing Discordance With Verbal Specificity

Both verbal specificity and gesture–speech discordance are, independently, significant predictors of which children will benefit from instruction. Thus it is important to examine how these two indexes are related.

[4] Although, in general, degree of explicitness or degree of vagueness does not seem to be critical in predicting learning outcome (compared with degree of specificity, which is related to learning outcome), we discovered an interesting anomaly in our data. In particular, the 2 children who were most vague (one had a mean specificity score of 0.39 and the other 0.44) did not succeed after instruction. If we had a larger sample (and had more than 2 subjects who on average mentioned fewer than 50% of the necessary elements to explain their solution strategies), we might have found that the relationship between specificity and learning outcome could be characterized with a bimodal distribution. In this way, children who do not mention even 50% of the necessary elements (i.e., are very vague) may be on the road toward an understanding of equivalence in the limited fashion exhibited by the children classified as explicit during the pretest. On the other hand, children who mention more than 50% but fewer than 100% of the necessary elements appear to be on the road toward an understanding of equivalence. Although we provide speculations with only 2 subjects, it is possible that extreme vagueness might indicate a lower, rather than higher, transitional level of understanding about equivalence.

[5] Specificity scores were not computed for the posttest problems containing equivalence explanations. This was done for two reasons. First, we chose not to compare, for example, explicit lack of understanding with explicit presence of understanding. Second, it was impossible to use the children's written posttest solutions to standardize the children's verbal explanations of the solutions, because there are many different ways that children could have derived an equivalence solution. This was not the case for the incorrect solutions children provided on the pretest, in which determining the particular strategy was transparent.

First, we correlated children's classification as discordant or concordant with their classification as vague or explicit. We found that these measures were not significantly related ($r = -.052$, *ns*). This means that the children who were classified as discordant were not necessarily classified as vague and children who were classified as concordant were not necessarily classified as explicit.

Next, we performed a log-linear analysis to determine the best model for considering how gesture–speech discordance and verbal specificity might be combined to predict learning outcome. The cell frequencies on which this model was tested are presented in Table 3. Results from this analysis suggested that an additive model fits the data quite well (likelihood ratio $\chi^2 = .369$, $p = .54$). Furthermore, the interaction term adds nothing significant to the fit of the model. Thus, both verbal specificity and discordance independently add to our ability to predict learning outcome, but the interaction of these two does not.

Discussion

The transitional knowledge state has been defined in many ways. What we have done in the present research is provide an index of this state. In the remainder of this article, we attempt to characterize a child who is in a state of transitional knowledge in light of what we have learned from the verbal specificity index. First, we attempt to explain why it was that children who were in a transitional knowledge state tended to provide vague explanations and why children who were not in a transitional knowledge state tended to provide explicit explanations. Next, we compare verbal specificity and gesture–speech discordance as indexes of transitional knowledge. Finally, we discuss the usefulness of indexes of transition.

What Does Verbal Specificity Represent?

Conceptual organization has been linked to receptivity to instruction and thus to the transitional knowledge state (e.g., Strauss, 1972; Strauss & Langer, 1970). Keil (1984) argued that learning depends on the structure and organization of one's current representations. In general, the more tightly the knowledge is organized, the less receptive the individual is to instruction, and the more loosely the knowledge is organized, the more receptive the individual is to instruction.

Relating the notion of conceptual organization to the present set of findings, we found that the children with explicit explanations were relying on tightly organized mathematical knowledge that works for similar-looking (addition) problems but not knowledge of mathematical equivalence. Children with vague explanations seemed to have an understanding that the knowledge and procedures that they possessed for similar-looking problems were not appropriate, but they had not yet discovered appropriate strategies. Although we cannot confirm why children would be vague when their knowledge is in transition, we suggest that the level of specificity indicates the degree to which information is organized. Thus, children who use explicit speech indicate that they have a relatively coherently organized understanding. Conversely, children who use vague speech indicate that they have a relatively disorganized understanding. The results lead us to infer that different levels of verbal specificity indicate a difference in the way information may be structured. Vague speech reflects unorganized knowledge, and the need for organization (or the sense that current knowledge is somehow not appropriate) permits instruction to have an impact on knowledge acquisition.

Another reason that children would be vague when their knowledge is in transition or explicit when their knowledge is not in transition may be found in theories about the constraints on working memory (Case, 1985). It may be that the explanations provided by vague children were condensed because these children simultaneously entertained the individual steps of the problem as well as other aspects of the problem (e.g., certain principles of mathematical equivalence). In contrast, it may be that the explanations provided by explicit children followed a step-by-step execution of production rules, without interference from other considerations. In other words, because of constraints in working memory and the need to focus on aspects of the problem that were competing for space, the vague children may have sacrificed explicit descriptions of production rules (see Siegler & Crowley, 1991, for a similar argument).

Comparing Indexes of Transitional Knowledge

We know that the transitional knowledge state can be indexed definitively in a post hoc analysis by whether or not a child has benefited from instruction. Both gesture–speech discordance and verbal specificity can be used to index a child's state of transitional knowledge prior to instruction because both can predict which children will benefit from instruction. Still, neither one is a perfect index. It is important to compare these two indexes (which are purportedly marking the same phenomenon). One might expect that if the indexes were tapping the same representation of knowledge, then they would correspond perfectly in terms of predicting who would and who would not benefit from instruction (and correlate perfectly

Table 3
Discordance and Specificity as Predictors of Benefiting From Instruction

	No. of subjects			
Learning outcome	Vague and discordant	Explicit and concordant	Vague and concordant	Explicit and discordant
Successful	4	1	5	4
Not successful	1	12	4	5

with each other). However, this is not the case. In fact, there was no systematic relation between gesture–speech discordance and verbal specificity.

There are at least three possible (but not mutually exclusive) explanations for the lack of overlap between these two indexes. First, the lack of overlap may have been due in part to error in the indexes' ability to determine correctly which children were in a state of transitional knowledge. It may be that both indexes were measuring the same aspect of a child's understanding of mathematical equivalence, but they had nonoverlapping measurement error. In other words, specificity and gesture–speech discordance may both pick up transitional knowledge but identify different children when they are incorrect at picking up transitional knowledge. While we realize that this is possible, it seems highly unlikely that the error terms of these indexes would be so highly and negatively correlated if they really measure the same quality.

Second, although both of these indexes are tapping children's states of transitional knowledge, the outcome measure may have been too stringent and the transitions that some of the children accomplished may not have been as advanced as what we recognized as an accomplished transition. In the acquisition of mathematical equivalence, there may be a number of small-stepped transitions that occur prior to full understanding of the concept. If this is true, the children who were classified, for example, as vague and did not benefit from instruction may have been in a state of transition with respect to some aspect of mathematical equivalence (such as the notion that both sides of the equation need to be reckoned with) but not in transition to full and complete understanding of equivalence as was measured here. Thus, it may be instructive to look for additional signs of transition to assess more accurately children's developing understanding of equivalence.

Third, the two indexes may have tapped somewhat different aspects of children's developing understanding of mathematical equivalence. We have argued here that verbal specificity measures the children's organization of knowledge. Recently, others (Alibali & Goldin-Meadow, in press; Goldin-Meadow, Nussbaum, Garber, & Church, 1993) have argued that gesture–speech discordance may be tapping multiple representations of a concept, one represented in speech and another in gesture. It may be true that children who are both vague and discordant have disorganized multiple representations and children who are both explicit and concordant have a single highly organized representation. The children who are vague and discordant apparently have very unstable knowledge of mathematical equivalence and are therefore very likely to benefit from instruction; the children who are explicit and concordant apparently have very stable knowledge of mathematical equivalence and therefore are not likely to benefit from instruction (at least the instruction that was used here). We infer from this that either presenting both vague verbal information and discordant gestures and speech or presenting both explicit verbal information and concordant gestures and speech may serve as an excellent index of whether a child is in a state of transitional knowledge.

However, the knowledge of the other children, for whom the indexes do not concur about transitional status, may be partly in transition and thus may be ready to benefit from instruction, but not "as ready" as children who are fully transitional. The results of the log-linear analysis confirm this interpretation and suggest that the indexes actually tap alternative forms of transitional knowledge. Future work, which explores yet other indexes of transitional knowledge, may shed more light on how knowledge in transition is best represented.

We have made interpretations about the meaning of vague and explicit utterances—and, more generally, about transitional knowledge—based on data from mathematical-equivalence novices, rather than from experts. Although we could generate some hypotheses about how experts present their understanding of a concept, the data reported here cannot address this issue adequately. We suspect that as children gain expertise in a concept, they may start by explaining this concept in a somewhat vague fashion. Presumably, vagueness at this point would represent a not-yet-solidified, relatively disorganized, new understanding. However, with increasing expertise, we suspect that children would explain the concept with explicit speech. Explicitness, at this point, would represent a well-organized understanding of the concept. However, as understanding and expertise continue to increase, children may become vague again when they attempt to explain the concept. At this point, vagueness would represent a relatively loosely organized understanding, presumably because the child has moved far along in his or her understanding and no longer has the well worked-out strategies that were available when he or she was concentrating time and energy on that concept. Although the research to investigate this hypothesis has yet to be conducted, we strongly suspect that the specificity of children's speech will continue to change as their knowledge of a concept continues to change. In fact, if verbal specificity does not reflect change during later points in development, we would conclude that this index is a limited, albeit useful, marker of knowledge in transition.

Usefulness of Indexes of Transition

One of the more tangible benefits of having an index of the transitional knowledge state is that instructors can determine which children are most likely to profit from instruction. In other words, once teachers have assessed who is ready to learn, they may time instruction accordingly.

Teachers may already respond appropriately to children in states of transitional knowledge without being aware of it. As Goldin-Meadow, Wein, and Chang (1992) and Perry, Ifcher, and Woolley (in press) have suggested, children who are in a state of transitional knowledge emit cues about their transitional status. Adults seem to be sensitive to these cues, but they may not be aware of them. The cues may alert the child's teachers, parents, and more advanced peers to adjust their input to the child so that the child receives crucial information for conceptual reorganization or resolution of conflict, much as Saxe, Guberman, and Gearhart (1987) have described. Thus, children may play a role in shaping their own learning environment by providing essential information to those around them through their production (or lack of production) of cues such as vague explanations. In this way, the cues that children emit when they are in transition (including producing vague explanations) may play a role in the mechanism of cognitive change.

Conclusion

Clearly, understanding the characteristics of the transitional knowledge state is crucial to understanding how children incorporate new knowledge and learn. Verbal specificity has been shown to be a good index of the transitional knowledge state. Furthermore, the data presented here are consistent with the view that when children are in a state of transitional knowledge, they may have unorganized knowledge or are working within strained working memory limitations with respect to their understanding of mathematical equivalence.

Although the research presented here has provided information about the signs of when conceptual change is likely to occur, that is, when children are in a state of transitional knowledge, we have not addressed how different factors may influence that change. We believe that the interaction between different types of instruction and transitional states of knowledge needs investigation. To understand how children acquire knowledge, one must understand the mechanisms that influence this, including the factors that influence children's ability to incorporate new information.

Finally, it had been asserted that the transitional knowledge state is one that can be influenced by instruction. In this study and others, influence was defined as making large shifts in problem-solving strategies. However, cognitive change need not be an all-or-none phenomenon. It may be important to view cognitive change from the perspective that individuals take many small steps in their acquisition of a concept. Thus, indexes of transition should be sensitive to these changes so that one can ascertain how it is that these changes take place.

It is clear that there is much to be explored in terms of children's acquisition of concepts. Broadening our understanding of transitional knowledge both in terms of what constitutes cognitive change and what influences that change will help us to understand and facilitate the acquisition of knowledge.

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(Appendix follows on next page)

Appendix

Problems Used in the Pretest, Posttest, and Transfer Tests

Pretest Problems

1. $4 + 6 + 9 = \_\_\_\_ + 9$
2. $7 + 6 + 5 = \_\_\_\_ + 5$
3. $3 + 7 + 9 = \_\_\_\_ + 9$
4. $3 + 6 + 8 = 3 + \_\_\_\_$
5. $2 + 5 + 9 = 2 + \_\_\_\_$
6. $7 + 4 + 2 = 7 + \_\_\_\_$

Posttest Problems

1. $3 + 7 + 5 = \_\_\_\_ + 5$
2. $8 + 4 + 3 = \_\_\_\_ + 3$
3. $2 + 8 + 6 = \_\_\_\_ + 6$
4. $5 + 4 + 7 = 5 + \_\_\_\_$
5. $2 + 7 + 9 = 2 + \_\_\_\_$
6. $6 + 5 + 3 = 6 + \_\_\_\_$

Transfer Problems

During the transfer test, which was conducted as an interview, the following problems were written on the blackboard, and the child was asked to solve the problems and explain his or her solution.

$7 + 2 + 9 = 8 + \_\_\_\_$
$4 + 6 + 3 = \_\_\_\_ + 7$
$2 \times 4 \times 3 = \_\_\_\_ \times 3$
$5 \times 2 \times 4 = 5 \times \_\_\_\_$

Each of the following two problems was written on the blackboard. The experimenter told the child that he or she would see some problems that a teacher gave the class and he or she would also see some of the answers the students gave. The child was asked to pretend that he or she was a teacher and to tell the interviewer whether each of the answers was correct or incorrect.

$4 + 6 + 3 = \_\_\_\_ + 3$

Bill put 13 in the blank. Is Bill correct? Why/Why not?
Susie put 16 in the blank. Is Susie correct? Why/Why not?
Johnny put 4 in the blank. Is Johnny correct? Why/Why not?
Mary Ann put 10 in the blank. Is Mary Ann correct? Why/Why not?
Sam put 4 + 6 in the blank. Is Sam correct? Why/Why not?

$5 + 2 + 8 = 5 + \_\_\_\_$

Jerry put 15 in the blank. Is Jerry correct? Why/Why not?
Jenny put 20 in the blank. Is Jenny correct? Why/Why not?
Wendy put 10 in the blank. Is Wendy correct? Why/Why not?

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