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How Many Do You See? The Use of Nonspoken Representations in First-Grade Mathematics Lessons

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Not all relevant instructional information comes in the form of spoken words. In the present study, the authors examine multiple modalities of nonspoken forms of representation—specifically gestures, pictures, objects, and writing—used by 3 teachers in 3 years of 1st-grade math lessons. Teachers frequently used visually based modalities of representation and tended to produce combinations of representational forms rather than isolated representations. There were individual differences in their preference for representation types. Teachers used representations to accompany important spoken terms and to respond to student confusion. With nonspoken representations, teachers conveyed information critical to the explanation of mathematical concepts. Students must attend to the visual as well as vocal means of expressing information to gain access to all of the information presented in mathematics lessons.

Each individual, from elementary school students to professors of mathematics, relies on symbols to represent abstract quantities and operations when communicating about mathematical ideas. Nonspoken media, including fingers, graphs, written symbols, and counting blocks, can be essential to give mathematical concepts visible embodiments as referents. However, mathematical concepts or ideas are not inherent in the object or symbol; to use these effectively, students must learn how to represent and interpret multiple representations of mathematical concepts.

The National Council of Teachers of Mathematics (NCTM; 1991) *Professional Standards for Teaching Mathematics* addresses an aspect of this issue by making recommendations about instructional tools for reforming and improving the way teachers communicate with their students. In particular, the *Standards* recommend that mathematical tools become a means to justify and stand for ideas, making connections between notations and other representations. Although we agree, we have questions about how this goal is achieved. In particular, we would like to know how teachers can use mathematical tools to enhance student learning and understanding of mathematical representations. Furthermore, the NCTM (1991, see Standard 5) suggested that the mathematics teacher "models and emphasizes mathematical communication using written, oral, and visual forms" (p. 95). Unfortunately, very little is known about how this is typically done; thus, the present investigation addresses how teachers model and emphasize nonspoken representational tools for their students.

We reviewed literature with respect to three interrelated issues to inform our investigation. First, we examined how multiple representations might be used to promote or detract from positive learning outcomes. Second, we addressed the particular forms of these representations. Third, we focused on the teaching and learning of place value as a context for examining these issues. This review led us to a set of questions, which we first lay out and then address in this study.

How Instruction With Multiple Representations Might Enhance Learning Outcomes

Researchers (e.g., Hiebert & Wearne, 1992; Lampert, 1986) have investigated the instructional advantages of using multiple representations in elementary mathematics instruction. Some of the interest in this topic stems from Dienes's (1960) proposal that multiple, iconic representations aid instruction by offering embodiments of abstract entities. Instruction is aided because the embodiments can help students develop rich understandings and connections to new concepts.

Beyond Speech

In general, researchers who have explored the use of multiple representations in mathematics instruction (e.g., Lesh, Post, & Behr, 1987; McCoy, Baker, & Little, 1996; Webb, Gold, & Qi, 1990) have noted that teachers rely heavily on representations other than those expressed in speech. Nonspoken means of repre-

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sentation include both constructed, planned tools such as concrete manipulatives, written symbols, and pictorial images as well as natural, spontaneous nonspoken behavioral representations such as those conveyed through gesture. Together, these representations may hold much power for student learning because external representations are "essential to virtually all mathematical insight and understanding . . . Interactions with external, imagistic representations are important to facilitating the construction of powerful internal imagistic systems in students" (Goldin & Kaput, 1996, p. 415).

What this suggests is that, when analyzing instructional information, analyses of speech alone can be insufficient because iconic symbolic forms are often used along with speech to help students build mathematical understanding (e.g., Goldin & Kaput, 1996; Pimm, 1987). Along these lines, both Baroody (1989) and Sowell (1989) outlined the need for considering the quality and appropriateness of concrete manipulatives in mathematics instruction and cautioned readers that these instructional tools have limitations as well as advantages. We agree, and we echo the call for more systematic investigation concerning the multiple modalities of communication in mathematics instruction. As one example, we do not even know how common it is for teachers to use nonspoken representations in their lessons.

Benefits of Multiple Representations in Mathematics Instruction

Dienes (1960) advocated that teachers and their students use multiple representations in mathematics and work with multiple embodiments of a concept. Both Goldenberg (1995) and Pea (1987) recommended instructing with physically dissimilar forms of the same entity (Dienes, 1960). According to Goldenberg, an evaluation of students' understanding requires evaluation of students' synthesis of multiple representations. Goldenberg cleverly pointed out that if a student worked with only one type of representation, errors in test performance could reflect difficulty with that particular representation, not necessarily the concept itself. Goldenberg presented excerpts of a student discussing functions, with and without graphing the functions, and found that working only with abstract symbolic representations can lead teachers to assume prematurely that a student understood; only by working with a graph did the student show he had amended his mental representation. In a similar vein, Lesh et al. (1987) noted that "students seldom work through solutions in a single representational mode" (p. 37). A student's difficulty with translations between representations can reveal misconceptions, a clear benefit of using multiple representations. Still, the discussion and resulting conclusion that teachers should incorporate multiple representations in their mathematics instruction begs the question of how teachers can equip their students for learning with multiple representations.

Student Processing Considerations for Teachers Instructing With Multiple Representations

Although the potential benefits for student learning are great, when teachers attempt to help students build bridges between multiple forms of representations, the instructional considerations for teachers and instructional designers increase in complexity. As a result of this increased complexity, students may find that instruction with multiple representations places demands on their mental processing and mental representation of concepts—demands that are not necessarily present with simpler instructional representations. To aid their students, teachers should consider the implications of how students process not just one form of visual information, but multiple forms (e.g., Carson & Bostick, 1988; Mayer & Sims, 1994), and incorporate these implications into their teaching.

Mayer (1997), in citing Tarmizi and Sweller (1988) and Sweller (1994), argued that coherence across representations aids learning and that coherence comes from the construction of connections between the learner's verbal and visual representations. In his study of students' comprehension of scientific explanations, Mayer found that students performed better if they saw verbal and visual information simultaneously than if they received verbal explanations only. Mayer concluded that instructors can aid students' mental representations by presenting the information in a "coherent" manner; in other words, simultaneously presenting information in verbal and visual modalities. If teachers use multiple forms of representation but present the information separately in time and space, students may not remember a previous representational form to compare with the current form. Thus, teachers must find a way to present some of their instructive representations together in a coherent manner to link the forms so that students can build successively richer mental representations.

Mayer's (1997) conclusions echo the recommendations made by the NCTM: As the leader of the classroom community, the teacher should exhibit multiple ways of representing problem information to the students. According to the NCTM (1989, 2000) *Standards*, teachers should help students learn to find and identify multiple representations. We now turn to an examination of teachers' instructional techniques when teaching with multiple forms of external representations.

Forms of Multiple Representations

We begin this section with an example from Leinhardt and Schwarz's (1997) analysis of a lesson led by mathematics educator George Polya. They commented that "the most salient characteristic of Polya's lesson is the profusion of models and representations used to develop a sense of the problem and to support the instructional explanation" (p. 395). Leinhardt and Schwarz described Polya's actions while leading the discussion, and transcripts of the videotaped lesson included many instances of Polya gesturing to a drawing or model, pointing to lines or sides of a plane, or holding up a model tetrahedron as he mentioned them. Leinhardt and Schwarz concluded that much of the mathematics that Polya described was encoded in the multiple forms that he used, including drawings, models, and gestures. As noted earlier, investigations have examined aspects of instructional materials such as drawings and manipulatives (e.g., Baroody, 1989; Sowell, 1989), but we point out that one component of Polya's instruction is more ephemeral and less well understood: gesture.

Teachers' Gestures With Multiple Representations

With their speech, teachers indicate and discuss the instructional tools they are using to symbolize information. In addition to speech and concrete instructional tools, teachers and students also communicate through gestures, including gestures to objects and symbols. Researchers (e.g., Alibali, Flevares, & Goldin-Meadow, 1997; Crowder, 1996; Goldin-Meadow, Kim, & Singer, 1999; Perry, Berch, & Singleton, 1995) have begun to address students' and teachers' gestured representations during math and science learning. In addition to research specifically addressing gestures that convey conceptual information, other researchers (e.g., Pimm, 1995; Yackel & Cobb, 1996) have offered anecdotal evidence of the important role of gesture in mathematics lessons. For example, Yackel and Cobb noted that students often used gesture to convey information about mathematical symbols and concepts. Likewise, Pimm described a sequence during which a teacher used gestures to call attention to symbols and steps in an activity. Pimm commented that the teacher's expression of ideas "may alert us to the ways we talk about what we are doing when working algebraically with symbols, and whether we miscue pupils by the language we use" (p. 95). The communication, including gestures, accompanying symbols may either convey to students how a symbol is linked to a concept or confuse them by failing to link the symbols to a concept.

Teachers' gestures have the potential to serve a crucial role in building bridges between words and external representations. Recent work (e.g., Goldenberg, 1995; Bickmore-Brand, 1993) described gesture's role in communication about mathematical concepts anecdotally and called for a systematic evaluation of gesture's representational functions. Although Goldin and Kaput (1996) stated that computer-based instruction holds promise to serve this function because "up to now there has been no way physically to link manipulatives that represent quantities, quantitative relationships, and (especially) actions on these to other more formal representational systems" (p. 417), teachers' gestures to and about other representations may already serve such a function. In the present study, we explored how gesture functions with other types of representation in mathematics instruction.

Much of the research that we have cited has recommended linking multiple nonspoken forms of information, but such research has failed to show how such linkages are formed in a typical lesson. In the present study, we investigated how teachers link forms of external symbolic representation, with special attention to gesture's role in linking forms of representation.

Place-Value Instruction With Nonspoken Representations

The instructional focus of the analyzed lessons in the present study was place-value numeration. This is a basic and critical concept for children to learn in the early elementary years because much later mathematics depends on an understanding of place value (e.g., Ma, 1999). The writers of the NCTM (1989) *Standards* acknowledged that "understanding place value is [a] critical step in the development of children's comprehension of number concepts" (p. 39). Pimm (1995) commented that place-value numeration "is not a property of numbers, but a property of some numeration systems" (p. 22), making it a challenging concept for many students to learn and, relatedly, for teachers to teach. The challenge, as Walkerdine (1988) put it, is to help "children to see the numbers expressed by two-digit numerals not as a unified value . . . but as a union of two separable values" (p. 161). The place-value numeration system offers further challenges to English-speaking students for whom number names do not provide a transparent indication of a number's value (e.g., Miller & Stigler, 1987).

As an example of how the instruction of concepts can be enriched with nonspoken representations, we turn to Walkerdine's (1988) report. She described how the teacher effectively communicated through her gestures the correspondence between her verbal labels and the to-be-counted bundles. The teacher's repeated gestures to symbols during the lesson helped to designate the relationship between the name of a quantity and its imagistic or concrete representation. Walkerdine further commented that "the teacher is constantly, and often literally, pointing out that a numeral in a particular place represents a different pile . . . hoping that this correspondence will be obvious to the children" (p. 170). Although the preceding episode presented rich details of instruction, Walkerdine (see also Yackel & Cobb, 1996) did not look systematically at how teachers communicated place-value information and how teachers' gesture functioned in instruction.

Rationale and Outline of the Investigation

In this investigation, we addressed the process by which teachers communicate conceptual information about place value to their students through the use of multiple modalities of symbolic representation. Although the authors of the NCTM (1991, 2000) *Standards* and many researchers (e.g., Goldenberg, 1995; Lampert, 1986) have called for the use of concrete, pictorial, and symbolic representations to aid student learning, no study to date has looked in detail at how teachers use these forms of representation to convey mathematical information. For instance, rather than merely cataloging the types of materials, we investigated what teachers did with these recommended materials through the use of multiple iconic symbolic representations.

Although the evidence that we have reviewed is consistent with advocating the use of multiple representations, we do not know how often and in what manner teachers use various forms of visible representations and how these representations may be linked. To address this, we observed 3 first-grade teachers over 3 years and analyzed their mathematics lessons to characterize the manner in which they used multiple modalities of representation. We began simply by asking the following question: Which symbolic forms are used, in what combinations, and how often are they used in instruction? In particular, we examined teachers' rates of using four nonspoken symbolic representational forms: gestures, pictures, objects, and writing.

In the 1st year of the investigation, none of the teachers used the same curriculum. In the 2nd and 3rd years of the investigation, the teachers used an NCTM (1989) *Standards*-based (1989) curriculum (i.e., *Math Trailblazers;* Wagreich et al., 1997). From this aspect of the study's design, we asked the following question: Did the teachers use representations similarly when using the same curriculum, especially when comparing their use of representations when using different materials? We realized that the representations in the 1st year, when teachers used relatively traditional curricula, may have differed markedly from those we observed when teachers used the NCTM *Standards*-based curriculum. In addition, although traditional mathematics materials, such as those used by teachers in the 1st year, rely heavily on written information, classrooms influenced by a *Standards*-based curriculum should incorporate relatively more use of other forms such as pictures and concrete embodiments, which would influence representation use based on those forms. We note that, alternatively, if differences are not found between years for the teachers, a teacher's method of presenting information to her students is a stable characteristic of the individual, unaffected by a change in curriculum and materials. We also conducted analyses to determine whether the teachers presented information in ways similar to each other or whether they differed in displaying information.

We realized that representations are not necessarily displayed in isolation. Thus, in addition to examining the rate of the individual types of representations displayed by teachers, we asked the following question: How are different types of presentations combined? The combinations of different representation types offer a way of linking multiple instantiations of a quantity, as recommended by the NCTM (1989, 1991, 2000). Given the importance placed on this by the NCTM, we expected that teachers' rates of combining different forms of representations would increase from the 1st year of observation to the 2nd year, when they began to use a curriculum modeled after the NCTM's (1989, 1991) recommendations. In our analyses of this problem, in part because of the importance of gesture in communicative processes (e.g., Engle, 1998; Fujimori, 1997), we paid special attention to the role that gesture plays when representations are combined.

Although our major focus is on nonspoken representations and their contribution to mathematics instruction, we also conducted an analysis that examined nonspoken representations in relation to speech. We reasoned that the reform-based curricular materials, if they embodied the NCTM (1989) recommendations, should serve as relatively better instructional tools than traditional materials for the concept of place value. If so, we could anticipate that the teachers would use the visible representations both more often with spoken explanations of important place-value concepts than with spoken explanations of less critical concepts and more often after the introduction of the reform materials than with the traditional materials.

In line with our consideration of potential changes after adopting the NCTM-based curriculum, we also investigated the accessibility of teachers' representations. We did this because the NCTM (1989) recommended that teachers should take advantage of physical space while modeling with multiple forms of representation. If students receive their teacher's nonspoken as well as spoken communication, they have their teacher's complete instructional message, from which they may draw links from one form to another and then build an understanding of the instructed concept. Alternatively, if a teacher should refer to small or obstructed representations, such as small counters resting on a desk, then students are barred from accessing the full meaning of the instructional content. Thus, we evaluated the visibility of nonspoken referents to determine whether the materials and the teachers' presentation of these materials were more likely to be visible and accessible to students in the 2nd and 3rd years of observation than in the 1st year.

Examining teachers' nonspoken as well as spoken instruction can provide a measure of the explicitness of the instruction. Investigations in the vein of this study may serve as a foundation for exploring the degree to which teachers have adopted the NCTM's philosophy that teachers should model mathematical concepts for their students.

Method

Data Source

The sample consists of 5 days of videotaped mathematics lessons for each of the 3 teachers in each of the 3 years, with one exception: Only 4 videotaped lessons were available for one of the teachers from the 1st year;[1] thus, 44 lesson videotapes have been analyzed for the present study. We coded the total amount of time that the teachers spent on the lesson for the day and that the teachers were clearly visible on camera and engaged in whole-class activities. The teachers' whole-class lesson time varied greatly, from 7.75 min to 59.45 min; the average lesson length was 33.20 min (SD = 12.12) for Teacher H, 24.60 min (SD = 8.72) for Teacher M, and 22.18 min (SD = 8.92) for Teacher W. Results are reported as representations per minute, to control for differences in amounts of whole-class instructional time.

Participants

All 3 teachers—Teacher H, Teacher M, and Teacher W—are female. Teacher H is African American, and her students are African American and live in an urban working-class neighborhood; both Teachers M and W, although at different schools, are European American, and their students are primarily Hispanic and live in urban working-class neighborhoods. The teachers ranged in their years of teaching elementary school from 3 years (Teacher M) to 13 years (Teacher W) to 26 years (Teacher H) at the beginning of the 3-year longitudinal observation.

Teachers were recruited for participation directly by their principals or by a representative of the curriculum development team. When teachers agreed to participate, their schools received a full set of reform curricular materials, but the teachers were not promised anything as a reward for their participation. The teachers were not sought out because of any special expertise or reputation. They were, in this way, typical urban elementary teachers.

The 3 teachers who were the focus of this investigation were the 3 who, from a larger sample of 8 teachers, participated in each of the 3 years of investigation. Five of the original teachers participated for fewer than 3 years due to personal circumstances (maternity leave), were moved to second grade along with their first-grade students, or did not choose to implement the reform curriculum. One remaining teacher was not included for analysis because she conducted her class in Spanish, which limited our ability to analyze her lessons in the same ways that we could analyze the English-speaking teachers' lessons.

Coding Procedures

Coding of Nonspoken Representations

Coding from videotapes of lessons concentrated on the teachers' use of four common nonspoken symbolic forms of representation (i.e., pictures, concrete objects, written symbols, and gestures) to convey content information in instruction. Pictorial, concrete, or written representations were assigned when the teacher created or displayed such a representation to students either by holding it up in the air, displaying it on an overhead projector, or operating on it such as by joining groups of counting cubes. Only gestures conveying mathematical content were coded. These included points to indicate another representation type or symbolic gestures (e.g., holding up three fingers for "three" or moving hands toward one another to convey "grouping" counting items). Regulatory and disciplinary gestures

[1] The 2nd day of Teacher W's 1st year of instruction was not videotaped because of equipment failure. Thus, for the 1st year, we have four rather than five data observations, making all analyses that include her Year 1 lessons unbalanced designs.

such as points to call on students and gestures for "wait" or "quiet" were not included in the analyses reported here (see Appendix A for coding examples).

Representations were coded as occurring either as a single representation or as part of a multiple-representational expression. A representational expression included all of a teacher's representations expressed with her speech about one idea and not interrupted either by a pause of more than 3 s or by student speech. A multiple-representational expression consisted of a combination of two or more different types of nonspoken representations, such as gesture and writing (see Appendix B for examples).

Coding of Spoken and Nonspoken Representations

To measure how the teachers' use of visually based representations supported their spoken instruction, we analyzed a subset of the teachers' speech for the presence of accompanying nonspoken representations with the first 10 and the last 10 times the teacher said "ten" or "tens" and "number" or "numbers" in the lesson. We chose the term *ten* because of its critical importance in conveying information about base-ten place value. We chose the term *number* for comparison with *ten* because it is a critical mathematical term but not particularly critical to understanding place value.

Visibility of Nonspoken Representations

We assessed the visibility for students of each modeled representation expression. For instance, if a teacher addressed the whole class while pointing to beans on a student's desk, many students, seated several feet away or with their desks facing away from the teacher, could not see the teacher's indications of the beans. We examined the frequency of teachers' representations judged as difficult to see; representations judged as nonvisible were omitted from other analyses.

Materials

Concrete instructional materials in the 1st year included beans, coins, popsicle sticks, small counting cubes, and the students themselves as the teachers grouped them as tens and ones. Teachers also created drawings of counting sticks and stars, one teacher referred to a "hundred chart" hung on the chalkboard, and one teacher used the overhead projector to display problems. In the 2nd and 3rd years of instruction, all the teachers used similar materials: beans, a game spinner, small counting cubes, and large counting cubes. In addition, teachers used an overhead projector to refer to data tables, ten-frame grids, and counting grids. They created drawings of quantities and referred to pictures such as drawings of animals to represent the numbers 1 through 10. All 3 teachers used the overhead projector to display information.

Reliability

An independent coder examined between 10% and 20% of whole-class instructional time in each of the 44 lessons. This coder examined the videotaped nonspoken representations (gestures, pictures, objects, and writing). Interrater reliability for coding representation types was assessed at .79, using Cohen's (1960) kappa (simple agreement = .86).

Results

General Notes and Descriptive Statistics

An alpha level of .05 was used for all statistical tests. Unless otherwise noted, multiple comparisons of means were conducted with the Dunn–Sidak procedure (see Sidak, 1967) for planned, pairwise comparisons. This procedure is robust for unequal numbers of observations and more powerful than Scheffe's multiple-comparison procedure.

All 3 teachers used nonspoken representations frequently in their instruction of place-value numeration. Across years, Teacher H used an average of 5.40 representations per minute (SD = 1.89), Teacher M used an average of 5.07 (SD = 2.38), and Teacher W used an average of 7.14 (SD = 2.56). In the next set of analyses, we explore how teachers used these representations and whether this usage changed or remained the same across the 3 years of investigation.

Comparisons of Representation Rates by Year, Teacher, and Type

We expected a difference between teachers' use of representational forms between the baseline year and the years of instruction with the NCTM *Standards*-based (1989) curriculum. Differences between teachers in rates of nonspoken representations would indicate significant individual differences by teacher and thus that some teachers' students received a higher rate of nonspoken information than did others.

The mean number of representations per minute for each teacher, for each year, is displayed in Figure 1. To test for overall differences in the rate of representations by teacher, year, and representation type, we performed a Teacher × Year × Representation Type three-way analysis of variance (ANOVA) and found a significant Representation Type × Teacher interaction, $F(6, 140) = 2.46$, $p = .027$. No significant effects resulted for overall rate differences by year, $F(2, 140) = .53$, *ns*, the Year × Teacher interaction, $F(4, 140) = 1.85$, *ns*, or the Representation Type × Year × Teacher interaction, $F(12, 140) = .90$, *ns*. Across years, Teachers H and W conveyed more gestures than did Teacher M, $t_{DS}(2, 45) = 2.59$, and $t_{DS}(2, 44) = 4.29$, respectively, Teachers M and W conveyed more object representations than did Teacher H, $t_{DS}(2, 45) = 3.17$, and $t_{DS}(2, 44) = 3.17$, respectively, and Teachers H and W conveyed more writing than did Teacher M, $t_{DS}(2, 45) = 3.79$, and $t_{DS}(2, 44) = 3.48$, respectively (see Figure 1).

In addition to the difference by representation type, there was a significant Teacher × Representation Type interaction in Year 1, $F(2, 44) = 7.14$, $p = .002$. Within Year 1, a comparison test revealed that Teacher W conveyed representations at a higher overall rate than did Teacher M, $t_{DS}(2, 44) = 3.47$, $p < .001$; that is, these two teachers differed in their style of conveying nonspoken information. The representation rate did not differ significantly by teacher, $F(2, 48) = .63$, *ns*, and $F(2, 48) = .40$, *ns*, for Years 2 and 3, respectively.

Visibility of Nonspoken Representations

We found that teachers frequently used nonspoken means to convey information, but in fact, the rates presented earlier do not include all of the representations conveyed by the teachers in their lessons because we excluded representations judged as not visible to a majority of students. Sometimes the teachers spoke about an entity that was out of view of the majority of the students; this occurred most often when a teacher referred to a representation on one student's desk, such as a set of counting cubes, without holding them up for the rest of the class to see. Agreement on the coding of visibility of judgments was high (simple agreement = .98).

A Teacher × Year, two-way ANOVA on the proportions of visible representations revealed a significant interaction for the differences in the judged visibility of teachers' nonspoken instructional representations, $F(4, 35) = 3.19$, $p = .025$. Note that we transformed the proportions with an arcsine transformation to remove the dependency between the means and variances (Kirk, 1995), a procedure we followed when we tested proportions in later analyses. To locate the sources of the significant interaction, we performed a one-way, by year ANOVA for each teacher. In the 1st year, an average of 80% of Teacher H's representations were judged as visible to the majority of her students; this rate increased to 96% in the 2nd year and to 100% in the 3rd year, $F(2, 12) = 5.38$, $p = .021$. Teacher H's representations were judged as more visible in the 3rd year than in the 1st year of observation, $t_{DS}(2, 12) = 3.28$, $p = .021$.

Likewise, Teacher M's presentations of information became more visible in the 2nd year of observation; in Year 1, 88% of her representations were judged visible to all students, and the proportion increased to 100% for each of the next 2 years, $t_{DS}(3, 12) = 3.60$, $p = .012$, and $t_{DS}(3, 12) = 3.43$, $p = .015$, comparing Year 1 with Years 2 and 3, respectively. Teacher W's representations were judged as at least 99% visible in all of her lessons in all 3 years, and changes across years were not significant, $F(2, 11) = 2.35$, *ns*. Thus, the 2 teachers whose representations were judged as difficult to see in the 1st year of observation displayed information in more visible ways when they switched to Math Trailblazers (Wagreich et al., 1997).

Comparisons of Representation Rates for Each Teacher

Then, examining only the representations judged as highly visible, we tested for differences in each representation type by year for each of the 3 teachers with a Year × Representation Type, two-way ANOVA with repeated observations by day for each teacher.

Teacher H

Across years, Teacher H used the representation types at significantly different rates, $F(3, 48) = 20.68$, $p < .001$ (see Figure 2). Across the 3 years of observation, she used more gesture than pictures, $t_{DS}(3, 48) = 6.84$, $p < .001$, objects, $t_{DS}(3, 48) = 6.33$, $p < .001$, and writing, $t_{DS}(3, 48) = 2.76$, $p = .047$, and more writing than pictures, $t_{DS}(3, 48) = 4.07$, $p = .001$, and objects, $t_{DS}(3, 48) = 3.57$, $p = .005$. No significant differences in representation type were obtained for either the main effect of year, $F(2, 48) = .05$, *ns*, or the Year × Representation Type interaction, $F(6, 48) = .24$, *ns*. Thus, across years, Teacher H indicated mathematical information with her gestures more often than with the other coded modalities of representation. This trend held within each of the 3 years (see Figure 2).

Teacher M

There was a significant main effect of year on Teacher M's rate of representation use, $F(2, 48) = 5.48$, $p = .007$, and also a significant main effect of representation type across years, $F(3, 48) = 10.17$, $p < .001$, but not for their interaction, $F(6, 48) = .48$, $p = .818$ (see Figure 3). Teacher M conveyed more total representations in Year 2 than in Year 1, $t_{DS}(2, 48) = 2.69$, $p = .035$, and more representations in Year 3 than in Year 1, $t_{DS}(2, 48) = 3.04$, $p = .015$. Across years, Teacher M conveyed more gesture than pictures, $t_{DS}(3, 48) = 5.29$, $p < .001$, and writing, $t_{DS}(3, 48) = 3.91$, $p = .002$, and more objects than pictures, $t_{DS}(3, 48) = 2.75$, $p = .049$. Thus, like Teacher H, Teacher M showed preferences by representation modality in her rate of displaying information; unlike Teacher H, Teacher M's overall rate of representations increased from Year 1 to Year 2, coinciding with her adoption of a reform-based curriculum (see Figure 3).

Teacher W

For Teacher W, there was a significant main effect of representation type across years, $F(3, 44) = 11.86$, $p < .001$, but no significant effect of year, $F(2, 44) = .41$, *ns*, or their interaction, $F(6, 44) = .97$, *ns* (see Figure 4). Teacher W used more gesture than pictures, $t_{DS}(3, 48) = 6.03$, $p < .001$, objects, $t_{DS}(3, 48) = 4.28$, $p = .001$, and writing, $t_{DS}(3, 48) = 3.23$, $p = .028$. Thus, Teacher W conveyed information significantly more often in gesture than in the other three representation modalities. This is true for each of the 3 years of observation.

Thus, for all 3 teachers, mathematically related gesture was the most frequent form of representation, but otherwise, the teachers displayed individual differences in their preferences for displaying information to students.

Combinations of Representation

In this section of the Results, we take a careful look at the ways in which representations were combined. First, we examine the frequency of combinations compared with the presentation of a single nonspoken representation. Second, we examine which forms of representation are used in combinations. Third, we take a look at combinations used that had three or four, rather than just two, representations.

Proportion of Representations in Combination

We asked how often teachers combined representations, asking if the forms usually occurred alone or as a part of a multiple-representation expression. We found that teachers frequently combined representational forms. On average, Teacher H used 2.03 ($SD = .92$) combinations per minute, Teacher M used 1.99 ($SD = 1.07$), and Teacher W used 2.63 ($SD = 1.28$). For each of the teachers, the proportion of representations in combinations averaged more than half of all nonspoken expressions: .65 ($SD = .22$) of Teacher H's representations, .70 ($SD = .13$) of Teacher M's representations, and .65 ($SD = .20$) of Teacher W's representations were in combination. There were no significant teacher, $F(2, 35) = .34$, *ns*, year, $F(2, 35) = 1.98$, *ns*, or interaction effects, $F(4, 35) = 1.37$, *ns*, for the proportion of representations in combination. Thus, the proportion of representations in combination and the rate of combinations did not differ significantly by year or teacher. Furthermore, all 3 teachers presented the majority

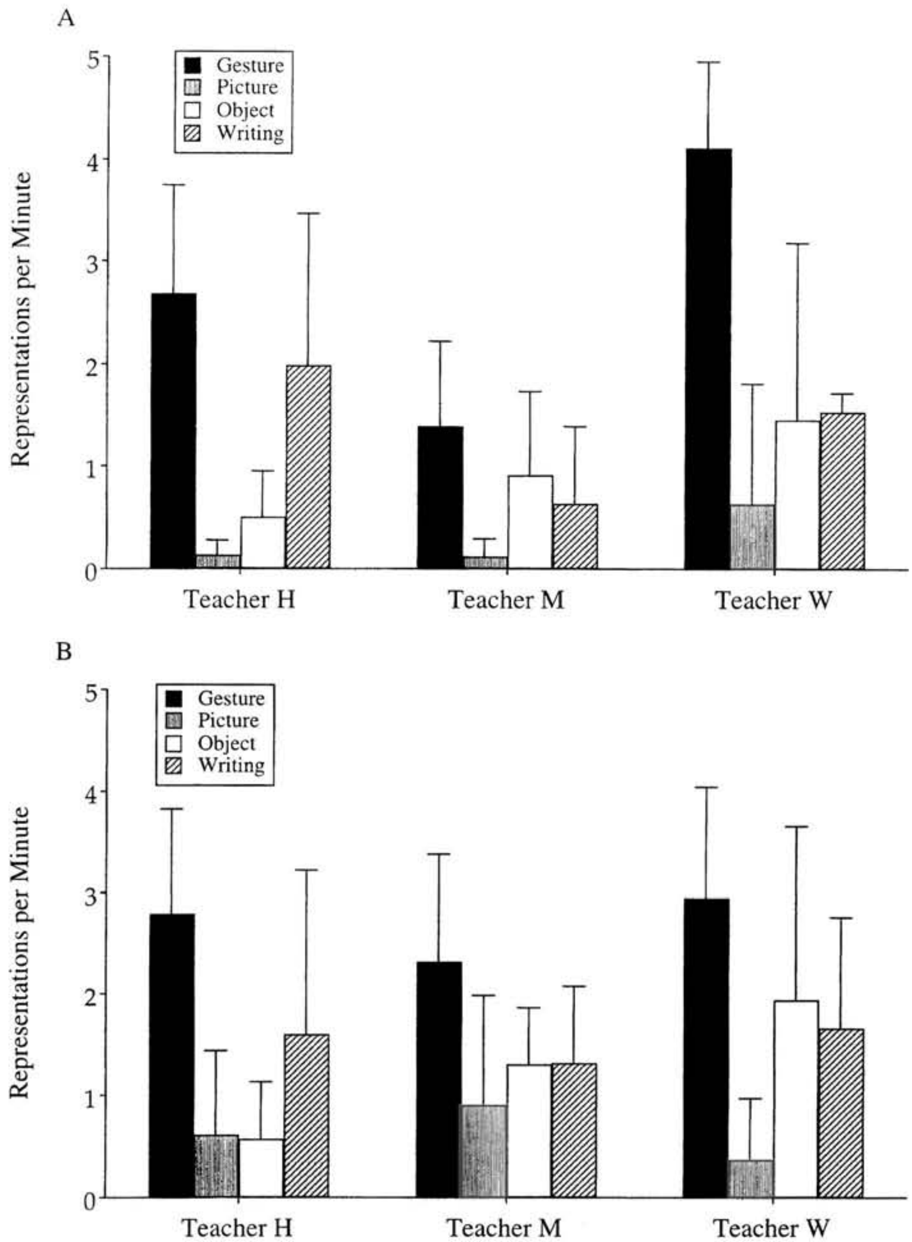

Figure 1. (A) Teachers' use of representations per minute in Year 1. Teacher W conveyed representations at a higher overall rate than did Teacher M. Error bars represent standard deviations from days of observation. (B) Teachers' use of representations per minute in Year 2. No significant differences were present by teacher. Error bars represent standard deviations from days of observation. (C [opposite]) Teachers' use of representations per minute in Year 3. No significant differences were present by teacher. Error bars represent standard deviations from days of observation.

of their representations in combination, which means that they linked forms of mathematical information, at least temporally.

Combinations That Included Gesture

Combinations almost always included a mathematical gesture. Across the 3 years, Teacher H used a mathematical gesture in 99% (*SD* = .002) of her combinations, Teacher M in 97% (*SD* = .04), and Teacher W in 99% (*SD* = .03); thus the teachers frequently used gesture along with the more traditionally studied nonspoken modalities to express mathematical information. This is potentially significant because gestures are produced spontaneously with speech, thus allowing teachers to simultaneously reinforce their spoken instruction with a visual representation (e.g., McNeill, 1992).

C

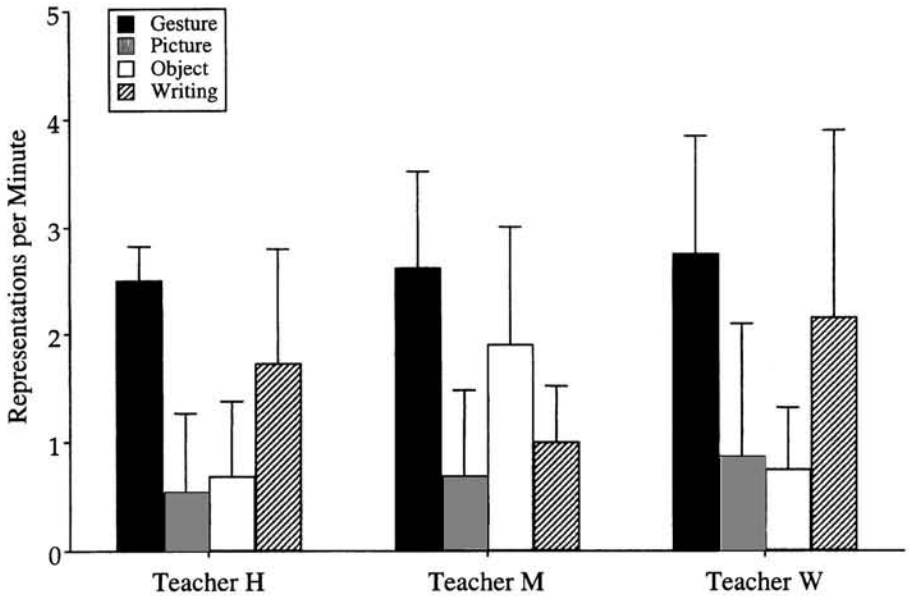

Figure 1 (*continued*)

Combinations With More Than Two Representation Types

A multiple-representation combination was coded when a teacher used two, three, or four different types of representations to express an idea. Using three or four different types of representation required the teacher to present and potentially integrate more displays of information than combining only two different types. Across years, .07 (SD = .07) of Teacher H's combinations had either three or all four types of representations, .15 (SD = .18) of Teacher M's combinations had either three or all four types of representations, and .17 (SD = .18) of Teacher W's combinations had either three or all four types of representations. There were no differences by teacher, $F(2, 35) = 1.25$, *ns*, year, $F(2, 35) = 1.49$, *ns*, or their interaction, $F(4, 35) = .85$, *ns*. Thus, the teachers did not differ in their display of these complex combinations, and they neither increased nor decreased their rates of such combinations from one year of observation to another.

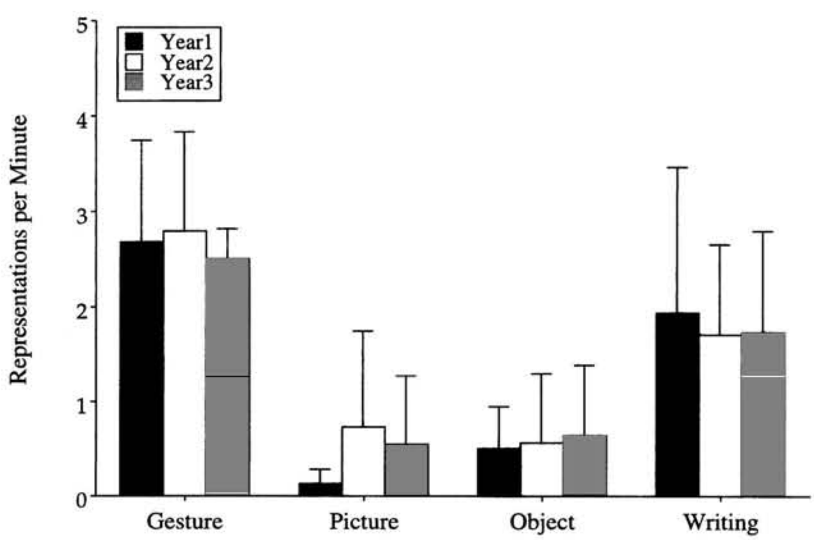

Figure 2. Teacher H's representation rates per minute across years. Across years, Teacher H conveyed more representations (n = 5) in gesture than pictures, more gestures than objects, more gesture than writing, and more writing than pictures and objects. No significant differences were indicated for any of the representation types by year. Error bars represent standard deviations from days of observation.

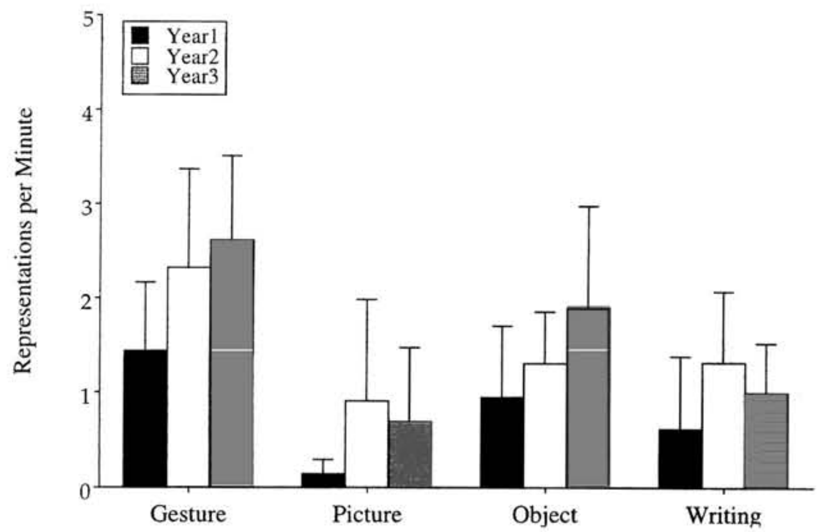

Figure 3. Teacher M's representation rates per minute across years. Across years, Teacher M conveyed more representations per minute (n = 5) in gesture than pictures, more gestures than writing, and more objects than pictures. Teacher M conveyed a higher overall rate of representations in Year 2 than in Year 1 and a higher rate in Year 3 than in Year 1. No significant differences were indicated for any of the representation types by year. Error bars represent standard deviations from days of observation.

The teachers produced a total of 313 combinations with three or four types. What did these combinations look like and what function might they serve? The 3 teachers conveyed only 15 four-type combinations, out of 2,592 total combinations. This low rate prompts us to ask the following questions: Why do they occur so infrequently? What are these four-type representations like?

The following transcript represents a typical example, taken from one of Teacher M's Year 2 lessons. The coded representation types are indicated in parentheses:

> Teacher M holds up seven unifix cubes (object) and then adds one more cube (object) to the stack. She then points (gesture) to the side of the third row (picture) of the 50 Chart grid and lays the cube stack down on top of the third row as she writes "28" (writing) in the 28th box on the 50 chart.

From this example, it is clear that the teacher must balance and coordinate multiple sources of information. Such a juggling act could be a demanding task both for her to present and for her students to process (Mayer, 1997; Sweller, 1994).

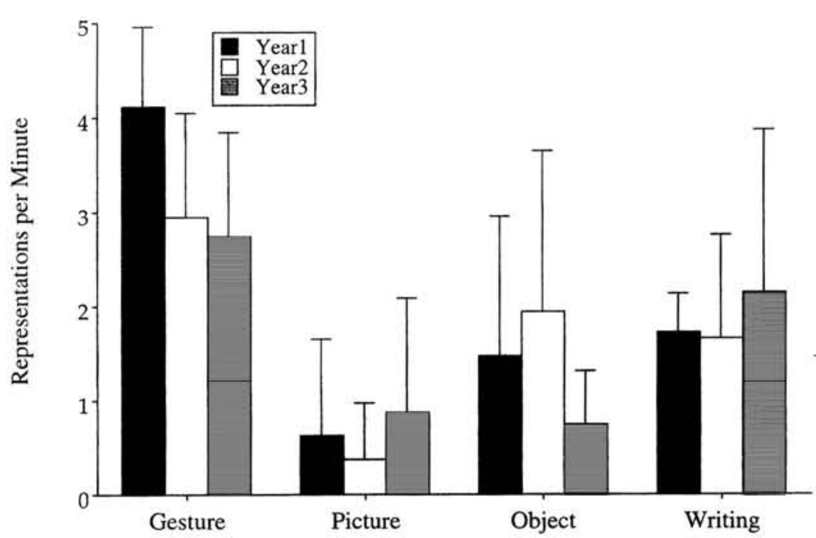

Figure 4. Teacher W's representation rates per minute across years. Across years, Teacher W conveyed more representations (n = 4 in Year 1, n = 5 in Years 2 and 3) in gesture than pictures, more gestures than objects, and more gesture than writing. No significant differences were indicated for any of the representation types by year. Error bars represent standard deviations from days of observation.

The combinations with three different types of representations, on the other hand, occurred much more often than did four-type combinations. Here is a typical example of a three-type combination, in which Teacher M pointed (gesture) to beans (objects) on a grid (picture) in one of her Year 2 lessons:

> Teacher M points (gesture) to the second ten frame (picture), which has two beans (object) in it, then points back up to the partially filled first ten frame (gesture, picture, and object), while explaining that the first set of ten must be completely filled before putting beans into the second.

In addition, it is worth noting that although the teachers used gesture in every lesson, they did not use each of the other representation types in every lesson. The teachers failed to use at least one, but sometimes two, of the other three types of representation in 17 of the 44 lessons, limiting the opportunity to observe a complex representation combination.

The Function of Nonspoken Representations

As we have just demonstrated, teachers frequently used nonspoken representations and representation combinations in instruction, but frequency alone does not explain their role in instruction or their value for student learning. To look at the function of these nonspoken representations, we examined their use in three ways. First, we examined the role of teacher-produced nonspoken representations before and after evidence of student confusion. In other words, we examined the effect of nonspoken representations in either producing or in remedying student confusion. Second, we examined nonspoken representations that accompanied teachers' speech during instruction. Here, we compared nonspoken representations that accompanied a mathematical term critical to the point of the lesson with those that accompanied an important mathematical term that was not particularly critical to the point of the lesson. Finally, we examined the lessons qualitatively for ways in which the nonspoken representations enhanced and augmented the teachers' verbal instruction.

Nonspoken Representations: Promoting or Remedying Student Confusion?

To investigate the role of nonspoken representations in learning, we looked at the evidence of impact on students during the lessons. We note that we chose to examine student and teacher behavior during the lesson. Devising a measure to be used after the completion of the lesson concerning the usefulness of any particular representation from a lesson would be difficult, if not impossible, because any particular representation may be preceded and followed by scores of others, thus interfering with an evaluation of any one representation.

To examine the question of the role of nonspoken representations in student confusion, we began by locating all verbal sequences in which teachers responded to student confusion and errors. From these, we traced teachers' use of nonspoken representations both before and after the noted confusion. A teacher could have used a nonspoken representation before confusion, immediately after the error or confusion, in her follow up to the student's correct response, or in any combination thereof.

We located 194 instances of student error or confusion that could be used for analysis. Two criteria were included: evidence of student confusion and visibility during instruction of the whole class. We found that the majority of instances of student confusion, 72%, were met with nonspoken representations in the teacher's immediate response to student confusion. Details about the number of instances of confusion and the proportion of these that were followed by nonspoken representations are displayed in Table 1. In general, we found that teachers responded to their students' confusion by producing a nonspoken representation.

Next, we examined whether the impact of nonspoken representations was to produce or to remedy confusion. For this analysis, we found that, of the 194 instances of student confusion, 53 contained nonspoken representations in immediate response to confusion when none had been used immediately preceding the confusion, but only 11 contained nonspoken representations before, but not immediately after, confusion. Thus, the teachers were almost five times more likely to respond to confusion with visual representations when they had initially omitted them than they were to omit them when they had initially used a visual representation; this was confirmed with a Wilcoxon signed ranks test, $z(n = 9) = 2.32$, $p < .05$, on the nonnormally distributed representations. This provides evidence that teachers were using nonspoken representations to remedy confusion.

In taking a careful look at these 194 sequences, we found no clear evidence existed for student learning in only 22 of them. In these instances, either the students did not offer a correct answer when asked, or the teachers did not repeat or rephrase the question that prompted the confusion. In the latter case, when the teacher did not directly address the confusion, the teacher explained or stated the correct answer without reasking her question. Thus, in those instances, we cannot be certain whether the students' confusion was immediately resolved, but in the other 89% of the instances ($n = 172$), the students did in fact have the opportunity to respond to—and actually responded correctly to—the teacher's reasking or rephrasing of the question. Taken together, teachers used nonspoken representations to remedy student confusion as judged by the students' ultimately correct responses.

As can be seen in Table 1, the only exception to teachers responding to student confusion in an overwhelming majority of instances was our observation of Teacher M's 2nd year. Indeed,

Table 1
Instances of Confusion and Proportion of Instances That the Teacher Responded With a Nonspoken Representation

Teacher and year	N	Proportion
H		
1	21	0.71
2	18	0.94
3	20	0.80
M		
1	16	0.75
2	10	0.30
3	23	0.91
W		
1	11	0.82
2	30	0.97
3	25	0.72

Teacher M seemed to be structuring her use of nonspoken representations differently during that year. By this we mean that we located a consistency in Teacher M's use of nonspoken representations following student confusion. In particular, in 60% of all instances of student confusion, Teacher M used nonspoken representations not immediately after the student's confusion but immediately after the student had produced a correct response; that is, Teacher M used her nonspoken representations to reinforce rather than scaffold the production of students' correct responses. Although this clearly deviates from the other teachers' and her own use of nonspoken representations during other years of observation, Teacher M's style of responding to student confusion in Year 2 also potentially serves an important pedagogical function.

The following example illustrates how nonspoken representations can help to bring about student understanding in ways that speech alone cannot:

> *Teacher W*: Susie, how many groups of ten do you have? (nonspoken representation: none)
> *Susie*: [No response] (nonspoken representation: none)
> *Teacher W*: How many groups of ten do you have that are all filled with the beans? (nonspoken representation: none)
> *Susie*: [No response] (nonspoken representation: none)
> *Teacher W*: How many groups of ten? (nonspoken representation: Teacher points [gesture] to a completed ten frame [picture] with beans [objects] on it)
> *Susie*: One. (nonspoken representation: none)
> *Teacher W*: One, that's right. (nonspoken representation: Susie writes *1* for the number of tens)

Teacher W's spoken communication first became more specific, but when Susie still did not respond correctly, Teacher W's nonspoken indications specifically addressed Susie's confusion. Although the present investigation focused on teachers' use of nonspoken, visually based forms of representation to convey meaning to their students, the coded sequences show how nonspoken information can supplement spoken information and indeed serve as the catalyst for resolving student confusion.

If representations are indeed potentially valuable for student learning, why do student errors and confusion sometimes follow them? We suspect that the answer to this question is that some instantiations of nonspoken representations may be more specific, more explicit, and thus more helpful than others. In our coding of responses to student confusion, we noticed that the teachers often appeared to become more precise in their actions, rather than doing a similar action or a less focused action. On closer review, we found that 55% of the instances of responses to student confusion that had some form of representation both preceding and following the confusion (n = 96) were judged as becoming more explicit in response to the confusion than were the representations expressed beforehand. This occurred significantly more often than did becoming less explicit, $z(n = 9) = 2.67$, $p < .01$. For instance, the following exchange took place in Teacher M's 3rd year, when counting beans in a ten frame:

> *Teacher M*: How many leftovers (ones)? (nonspoken representation: Teacher points quickly toward the eight leftover beans)
> *Juan*: [No response]
> *Teacher M*: How many leftovers? (nonspoken representation: Teacher points one by one to the leftover beans)
> *Juan*: Eight
> *Teacher M*: Eight (nonspoken representation: Juan writes the number 8 on the overhead)

The teacher's actions became more specific and less ambiguous about the quantity being discussed when she was responding to Juan's failure to answer. Her actions when repeating the question appear to disambiguate for Juan the term *leftover*. This example is typical of the instances of elaboration we observed in response to confused, incorrect, or hesitant students. Thus, in many instances, the teacher's initial question was accompanied by no actions or vague actions such as gestured points far from the referents, but her subsequent spoken response to student confusion was yoked to more specific actions. We recommend that future investigations further characterize the representations for the explicitness of mathematical content. Finally, it should be noted that in quite a few of the cases in which the teacher did not accompany her own response with an action, she prompted a student to act, for instance, by writing the number of tens on the board.

Nonspoken Representations Accompanying Speech

To gain a sense of the rate of nonspoken representations with important instructional terms in speech, we examined the frequency with which teachers used nonspoken forms when they said "ten." Because teachers varied in their use of this term across lessons, but each teacher used this term at least 20 times in each lesson, we coded the rate of nonspoken representations from each teacher's first ten and last ten utterances about "ten" while she was on camera and engaged in whole-class activities. The teachers frequently displayed information through nonspoken means as well as spoken means; saying "ten" was accompanied by a nonspoken representation, Ms = 10.07 (SD = 3.81), 10.67 (SD = 4.24), and 11.62 (SD = 3.24) across each of the 3 years, respectively. Thus, on average, teachers accompanied their speech about "ten" with visually based referents more than half the time. There was a significant interaction between teacher and year, $F(4, 35) = 11.00$, $p < .001$ (see Figure 5). Teacher H significantly decreased her use of accompanying nonspoken representations after Year 1, $t_{DS}(2, 12) = 2.88$, $p < .01$; Teacher M increased significantly each year, $t_{DS}(2, 12) = 3.31$, $p = .020$, comparing Year 1 and Year 2, and $t_{DS}(2, 12) = 3.02$, $p = .033$, comparing Year 2 and Year 3; and Teacher W did not change significantly over the 3 years of observation, $F(2, 11) = .64$, ns. The introduction of the new curriculum in Year 2 did not uniformly alter the teachers' use of nonspoken representations with the word *tens*; instead, the teachers showed individual differences in their use of visual means to supplement this term.

What are the teachers' presentations of *tens* like? That is, how do the spoken and nonspoken representations function together for a teacher to convey explanations or ask questions about base-ten place value?

The following example comes from one of Teacher M's Year 2 lessons:

> *Teacher M*: How many more does she need to make ten? (nonspoken representation: Teacher points [gesture] to a ten frame [picture] containing eight beans [objects])

The following example comes from one of Teacher W's Year 2 lessons:

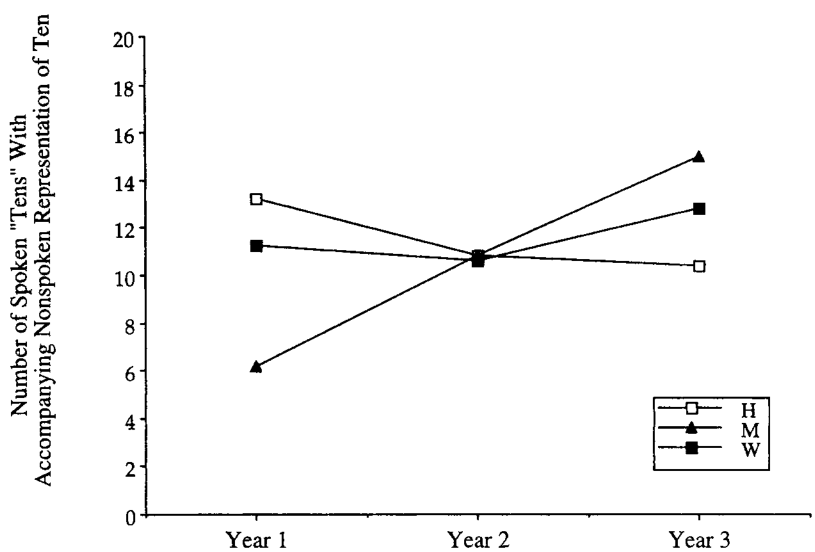

Figure 5. Teachers' rate of nonspoken representations accompanying speech about "tens." Teacher H's data show a significant teacher by year interaction in which Teacher H decreased her rate of representations accompanying "tens" speech, whereas Teacher M's data show the opposite trend. The sample consists of 20 observations for each teacher in each year. H = Teacher H; M = Teacher M; W = Teacher W.

> *Teacher W*: Here's another ten. (nonspoken representation: Teacher holds up and points to a stack of ten unifix cubes [objects])

These examples illustrate how teachers use visually based, nonspoken forms of information to indicate the idea of a "ten," a fundamental idea in base-ten place value. As a contrast, we analyzed the teachers' use of nonspoken forms of representation with the term *number*, which, like *tens*, is a noun that refers to quantity and occurred frequently in the lessons. Unlike our analyses of representation use with "tens," we found no differences by year, $F(2, 36) = 0.897, p > .05$, or teacher, $F(2, 36) = 0.649, p > .05$, in the rate of representations accompanying "number." Comparing the proportions of instances of verbal "number" and "tens" accompanied by representations in each lesson revealed that teachers used representational forms more often with "tens" than with "number" ($t = 4.73$, $p < .001$). Thus, the teachers appeared to privilege the term *tens*, a term central to the concept of place value, by using nonspoken representations to give meaning to the term.

Use of Nonspoken Representations to Enhance the Richness of Speech

As we examined the transcripts and videotapes, we were struck by the teachers' consistent reliance on nonspoken forms to communicate important conceptual and procedural information. To capture this, we reviewed the videotapes, along with transcripts of spoken and nonspoken communication, and mapped the relationship between the information that the teachers were presenting across modalities.

First, we examined cases in which conceptual information was being communicated. By conceptual information, we mean information about place value, especially the significance of tens versus ones. When examining the speech about conceptual information, we were struck by the ways in which the nonspoken modalities enriched or disambiguated the speech.

As a first example, we found that the nonspoken information was basically redundant but still enriched the information communicated in speech. In the example that we present here, one teacher's speech in expressing what she meant by grouping into tens versus counting by ones was clarified by her gestures. At the beginning of a 30-s segment, she said, "It was easy to group them in tens," while making a grouping gesture of scrunching her two hands together three times, in different places. Here, the teacher was demonstrating just how easy it was to push the beans together in (three) groups of tens.

Just seconds later, the teacher asked, "Did you ever lose count when you were counting by ones?" A student replied, "Yeah," and the teacher went on to say, "Yeah, I saw you were counting by ones" while demonstrating with her index finger how someone might point to three individual objects to keep track of the number (of objects) while counting. (This appeared to be a demonstration of how the students typically counted objects.) In the very next phrase, the teacher said, "And then you kind of forgot where you were [no accompanying gesture] followed by, "So I think that might have been a problem with you [the teacher's hand jerks around, as if lost]." In this 30-s example, the teacher was able to indicate that counting by ones can leave the student lost and that, if the student scoots beans together, he or she has less of a chance of leaving out a to-be-counted bean. Much of this information was available in speech alone, but that information was enhanced by the teacher's gestures that accompanied her speech. We found this sort of example to be abundant as teachers explained functional and conceptual aspects of place value.

As a second example, we found that the nonspoken information clarified or disambiguated the speech. For example, a teacher was trying to explain where to write the number of tens and where to write the number of ones on a worksheet. To communicate this to the students, she carefully read words from the worksheet—"Numbers of groups of ten"—but then pointed directly to the empty column beneath those words, thus indicating exactly where the students should write the number of tens. We note that this action further specified the information for students who were new to reading as well as mathematics.

In each of the examples, whether communicating to students about mathematical concepts or about how to complete their work, teachers used nonspoken representations to enhance their speech. In this way, students were given a richer sense of the mathematics (and how to succeed in math class) than if they had been provided with speech only. Our point is not that the sum total of all nonspoken expressions is sufficient to impact student understanding; instead, we see the nonspoken communication in classrooms as significantly adding to the ongoing verbal discourse. In particular, we expect that nonspoken representations add both clarification and richness to the spoken discourse (Glenberg & Robertson, 1999; Goldin-Meadow et al., 1999; Perry et al., 1995).

Discussion

The present investigation provided measures of the complexity of teachers' communication to students in first-grade place-value lessons. Examining only a teacher's verbal input to students creates an incomplete picture of the information available for students to build mathematical understanding. Analyzing lessons for the presentation of nonspoken symbolic forms gave an index of the richness and explicitness of teachers' instruction beyond speech. Thus, the analyses offered a deep look at how teachers used multiple nonspoken forms and modalities of representation to teach fundamental concepts and show connections between representations and within concepts.

We have three caveats and seven concluding points. Our first caveat is that although the present investigation provided a catalog of how information was conveyed by 3 teachers in mathematics classrooms, it concentrated on the modalities of presentation and how teachers combined these modalities to express information. The study did not systematically examine the content that teachers conveyed through multiple modalities of representation. Future investigations will look in detail at the mathematical content, such as the quantity or mathematical operation indicated by teachers and students through nonspoken modalities of representation, to determine more specifically how mathematical concepts are conveyed. For example, perhaps teachers tend to indicate quantities with objects but mathematical operations with writing. These future investigations, looking closely at both the teachers' and students' use of nonspoken representations, will further elucidate how teachers and students express ideas to each other about their mathematical knowledge.

Second, we caution that because the present investigation consisted of a longitudinal case study of only 3 teachers' use of nonspoken representations, the generalizability of the specific numeric results is limited. Optimally, we would conduct an investigation with more teachers and continue until no new insights were produced. The present study initiated this process by providing a framework for examining how teachers express instructional information through multiple modalities of nonspoken representations in mathematics instruction.

Third, the visually based forms of representation may have an essential role in conveying mathematics to students, but just as spoken words cannot teach if they are not heard, the nonspoken forms can only convey an instructional message to students if the students attend to them. Recent research (Goldin-Meadow et al., 1999; Perry et al., 1995) has shown that students attend to instructional gestures, and our analyses of teachers' responses to student confusion suggest that the students also attended to the teachers' instructional communication with objects, pictures, and writing. Future work should seek to investigate further how students attend and respond to teachers' instructional actions.

From the results, we draw seven conclusions about the teachers' use of nonspoken, visually based means to convey information to their students. First, teachers frequently use nonspoken modalities of representations to attach meaning to their spoken instruction of mathematics. Students received, on average, between 5.07 and 7.13 different nonspoken representations per minute (almost one every 10 s!), which offered a wealth of information beyond the teachers' spoken instruction. Clearly, such an abundance of instructional information should not be ignored when analyzing communication in mathematics classrooms.

Working with the new curriculum and its materials had a highly positive impact on the visibility of nonspoken representations for 2 of the observed teachers. This result may in part be an effect of the curriculum, which includes some large counters but, more importantly, encourages teachers to take advantage of physical space and display information on the chalkboard or an overhead projector. The change in curriculum and materials coincided with changes in the way Teachers H and M displayed information. In Year 1, Teachers H and M frequently indicated objects or drawings resting on only one student's desk or otherwise out of general view, inadvertently preventing many students from seeing what they indicated; they almost never did so during the 2nd and 3rd years of instruction. Consequently, the introduction of the new curriculum made the ways these teachers displayed information more public and accessible to their students.

Third, although we found a significant effect of year on Teacher M's overall rate of nonspoken representations, across the four representation types, the other 2 teachers did not significantly change the rate at which they displayed nonspoken representations when they began using the NCTM (1989) *Standards*-based curriculum in Year 2. Thus, the change in curriculum did not significantly change the overall rate at which those 2 teachers displayed the four modalities of representation to their students. In Year 1, Teacher W used more representations overall than did Teacher M. The lack of difference between teachers in Years 2 and 3 results in part from Teacher M's increase in her overall rate of nonspoken representations from Year 1 to Year 2. By increasing her overall rate of representations from Year 1 to Year 2, Teacher M came to resemble the other 2 teachers. What this suggests is that teachers may not change their rates of nonspoken representations in response to the introduction of a new curriculum, even one that emphasizes using manipulatives and other nonspoken forms, unless they initially had low rates of representation use.

Fourth, and relatedly, none of the teachers significantly changed the rate at which she displayed a given representation type from year to year, suggesting that, in spite of changes in curricular materials and activities, the teachers may each have a style for displaying each modality of representation. For example, although all teachers displayed gesture at a higher rate than the other modalities, Teacher H conveyed significantly more representations in writing than in pictures, and Teacher M conveyed significantly more objects than pictures across all 3 years of observation.

Fifth, as we just mentioned, in each of the 3 years, the teachers used more gesture than any of the other representation types; thus, the teachers used gesture, a form of communication that is typically spontaneous and produced simultaneously with speech, to indicate information more often than the more conventional and frequently studied instructional tools of pictures, objects, and writing. This suggests that researchers should pay attention to gesture and the information it conveys if they want to understand classroom communication in all its richness.

Sixth, the teachers frequently combined nonspoken representations, but the rate at which they did so did not change with the introduction of the new curriculum. Thus, the teachers linked forms of representation and modalities regardless of the lesson materials and activities. Their representation combinations almost always included mathematically relevant gestures; thus, gesture, which typically occurs naturally integrated with speech (e.g., Alibali, Kita, & Young, 2000; McNeill, 1992), appears to play an essential role in linking forms of information in face-to-face instructional settings. Although previous work by Mayer (1997) and Sweller (1994) has indicated that students may become cognitively overloaded by presentations of information, no study has previously looked at the effect of combinations of representations in teachers' naturalistic instruction. From the present investigation, we cannot draw strong conclusions about the impact that receiving a combination of three or four, as opposed to two, representation types has on student learning. Perhaps observing only two different modalities in combination is easier for students to process (and for teachers to produce) than representations presented in three or four modalities, but three or four different modalities may provide the basis for a richer and more flexible mental representation. Likewise, perhaps some ways of linking three or four different representation types are more effective than others. In addition, the clarity of combinations of representations may depend on the learner's current knowledge state with respect to the instructed concept and familiarity with the representational modalities. Future investigations will need to isolate different combinations of representations to identify which combinations are the most effective when introducing new concepts to students and to determine whether these multiple modalities offer an integrated "composite signal" (Engle, 1998).

Seventh, and finally, the nonspoken representations supply an essential but incomplete depiction of classroom communication. Our analyses of teachers' use of nonspoken representations with speech present important information about the way these communicative forms function together. We found that the teachers often used visual forms of representation to respond to their students' confusion, and the form of the teachers' response was often sensitive to the confusion. For example, the teachers' actions often became more specific and focused while their speech sometimes merely repeated the exact same question. Analyses confined to spoken classroom discourse miss crucial aspects of instructional information that teachers present visually.

Relatedly, our analysis of teachers' use of nonspoken representations accompanying speech about "tens," a key term in place-value instruction, also addressed how the teachers supported speech with nonspoken forms. Teachers often invoked nonspoken forms of representation while verbally indicating key terms in place-value numeration. Teachers' nonspoken representations functioned to assign and clarify mathematical meaning for their students.

In sum, the analyses of teachers' representations from the 3 years of instruction presented here offer a deep examination of the process of teachers' communication, beyond solely the teachers' words; it represents a step toward understanding the processes of instruction as teachers aid students in constructing mathematical knowledge. To appreciate the multiple forms of communication in mathematics classrooms, we must observe not merely with our ears but also with our eyes.

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Appendix A

Examples of Representation Coding

Representation type	Examples
Gesture	1. Teacher holds up index finger while saying "that's one ten." 2. Teacher cups both hands toward each other in the air while saying "you have a group of ten beans." 3. Teacher points with index finger to each of ten blocks in a stack of ten cubes while counting them.
Picture	1. Teacher draws a set of ten beans on an overhead. 2. The teacher puts an overhead with ten previously drawn beans onto the projector.
Object	1. The teacher snaps together two sets of five large unifix cubes. 2. The teacher forms piles of ten beans on the overhead projector.
Writing	1. The teacher writes a number on the board to indicate the cubes counted by a group of students. 2. The teacher holds up a card indicating a number.

Appendix B

Examples of Each Type of Representational Combination

Representation combination	Example
Gesture and picture	The teacher holds up ten fingers and then points to her drawing of ten beans.
Gesture and object	The teacher holds both hands flat apart, one above the other, to represent a stack of cubes and then holds up a stack of cubes.
Gesture and writing	The teacher holds up five fingers then writes a "5" for tens on the board.
Picture and object	The teacher places a bean within a box on an overhead grid and then traces around the perimeter of the box while saying "this is the tenth bean."
Gesture, object, and writing	The teacher holds up a set of five cubes, then with the index finger of her other hand points from the cube stack to a five written on the overhead.
Gesture, picture, and object	The teacher holds up a stack of ten cubes and then points to a row of ten boxes on the overhead grid saying that ten cubes will fill up a row.
Gesture, picture, and writing	The teacher points to eight circles drawn to represent beans on the overhead while counting them and then writes "8."
Gesture, picture, object, and writing	The teacher puts six beans into each of the six empty boxes of a ten frame on the overhead; then with her pen in the air above the boxes she traces the outline of the ten frame and then writes the number of ten frames completed.

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